Physics 8.02 **Exam Two Mashup** **Fall 2004**

Some (possibly useful) Relations:

$$\oiint_{\text{closed surface}} \kappa \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{Q_{\text{inside}}}{\varepsilon_o}$$

$d\vec{\mathbf{A}}$ points from inside to outside

$$\Delta V_{\text{moving from } a \text{ to } b} = V_b - V_a = -\int_a^b \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}$$

$$-\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = 0$$

$$V_{\text{many point charges}} = \sum_{i=1}^{N} \frac{q_i}{4\pi\varepsilon_o \left|\vec{\mathbf{r}} - \vec{\mathbf{r}}_i\right|}$$

$$C = \frac{Q}{\Delta V} \qquad \Delta V = \frac{Q}{C}$$

$$C_{\text{parallel plate}} = \frac{\varepsilon_o A}{d}$$

$$U = \frac{1}{2} C \, \Delta V^2 = \frac{Q^2}{2C}$$

$$\frac{1}{C_{\text{series}}} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$C_{\text{parallel}} = C_1 + C_2$$

$\vec{\mathbf{E}} = \rho \vec{\mathbf{J}}$ where ρ is the resistivity

$\Delta V = i R$

$$R = \frac{\rho L}{A}$$

$$\frac{1}{R_{\text{parallel}}} = \frac{1}{R_1} + \frac{1}{R_2}$$

$R_{\text{series}} = R_1 + R_2$

$$P_{\text{ohmic heating}} = i\Delta V = i^2 R = \frac{\Delta V^2}{R}$$

$$\vec{\mathbf{B}} = \frac{\mu_o}{4\pi} \frac{q\,\vec{\mathbf{v}} \times \hat{\mathbf{r}}}{r^2} \quad \left|\vec{\mathbf{v}}\right| << c \qquad d\vec{\mathbf{B}} = \frac{\mu_o I}{4\pi} \frac{d\vec{\mathbf{s}} \times \hat{\mathbf{r}}}{r^2}$$

$\hat{\mathbf{r}}$ points **from** source **to** observer

$$\oint_{\text{contour}} \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \mu_o I_{\text{through}}$$

where I_{through} is the current flowing through any open surface bounded by the contour:

$$I_{\text{through}} = \int_{\text{open surface}} \vec{\mathbf{J}} \cdot d\vec{\mathbf{A}}$$

d**s** right-handed with respect to d**A**

$$\oint \mathbf{E} \cdot \mathbf{ds} = -\frac{d}{dt} \int \mathbf{B} \cdot \mathbf{dA} \qquad \varepsilon = -N \frac{d\Phi_{\text{sgl loop}}}{dt}$$

$$\vec{\mathbf{F}} = q\,\vec{\mathbf{v}} \times \vec{\mathbf{B}}_{ext} \qquad d\vec{\mathbf{F}} = I\,d\vec{\mathbf{s}} \times \vec{\mathbf{B}}_{ext}$$

$$F_{\text{cent.}} = mv^2/r$$

$\vec{\boldsymbol{\mu}} = I A \hat{\mathbf{n}}$ $\quad$ $\hat{\mathbf{n}}$ perpendicular to loop, right-handed with respect to I

$$\vec{\boldsymbol{\tau}} = \vec{\boldsymbol{\mu}} \times \vec{\mathbf{B}} \qquad F_z = \mu_z \frac{dB_z}{dz}$$

Cross-products of unit vectors:

$$\hat{\mathbf{i}} \times \hat{\mathbf{i}} = \hat{\mathbf{j}} \times \hat{\mathbf{j}} = \hat{\mathbf{k}} \times \hat{\mathbf{k}} = 0$$

$$\hat{\mathbf{i}} \times \hat{\mathbf{j}} = \hat{\mathbf{k}} \quad \hat{\mathbf{j}} \times \hat{\mathbf{k}} = \hat{\mathbf{i}} \quad \hat{\mathbf{k}} \times \hat{\mathbf{i}} = \hat{\mathbf{j}}$$

Some potentially useful numbers

$$k_e = \frac{1}{4\pi\varepsilon_0} = 9 \times 10^9 \frac{\text{N m}^2}{\text{C}^2} \qquad \mu_o = 4\pi \times 10^{-7} \frac{\text{T m}}{\text{A}}$$

Breakdown of air	$E \sim 3 \times 10^6$ V/m
Earth's B Field	$B \sim 5 \times 10^{-5}$ T = 0.5 Gauss
Speed of light	$c = 3 \times 10^8$ m/s
Electron charge	$e = 1.6 \times 10^{-19}$ C
Avogadro's number	$N_A = 6.02 \times 10^{23}$ mol^{-1}

Problem 1 (25 Points)
Circle your choice for the correct answer to the five questions below.

A: A particle with known charge q and known mass m is performing circular motion in a uniform magnetic field of known magnitude B (as usual, the orbit is perpendicular to the magnetic field).

With q, m, and B known, examine the following statements:
(a) The radius R of the orbit can be determined uniquely.
(b) The speed v of the particle can be determined uniquely.
(c) The angular velocity ω of the particle can be determined uniquely.

Circle the ***one correct statement*** below

1. Only (a) is correct.
2. Only (b) is correct.
3. Only (c) is correct
4. Only (a) and (b) are correct.
5. Only (a) and (c) are correct.
6. Only (b) and (c) are correct.
7. All are correct.
8. None is correct

B: Consider an infinitely long cylindrical solenoid of radius R with n turns per unit length and a current I. The magnetic field due to this solenoid satisfies:

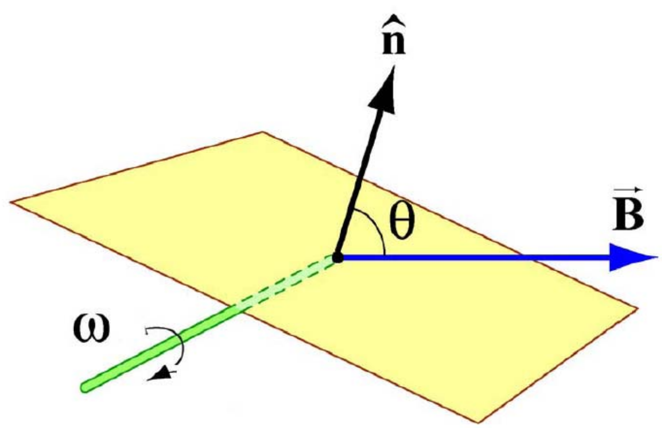
1. $B = 0$ for $r < R$ and $B = \mu_0 nI$ for $r > R$.
2. $B = \mu_0 nI$ for $r < R$ and $B = 0$ for $r > R$.
3. $B = \frac{1}{2}\mu_0 nI$ for $r < R$ and $B = 0$ for $r > R$.
4. $B = 0$ for $r < R$ and $B = \frac{1}{2}\mu_0 nI$ for $r > R$.
5. $B = \mu_0 nI$ for all r.

C: Consider a charged circular loop of radius R and linear charge density λ (charge per unit length) glued down to the loop. Suppose the loop is rotating with angular velocity ω about the axis normal to the loop and going through its center. The loop acts as a magnetic dipole with magnetic dipole moment μ given by

1. $\mu = \lambda \pi R^2$
2. $\mu = \lambda \omega \pi R^2$
3. $\mu = 2\lambda \omega \pi R^2$
4. $\mu = \lambda \omega \pi R^3$
5. $\mu = 2\lambda \omega \pi R^3$

D: An infinitely long wire carries a current I. What is the magnitude B of the magnetic field a distance r away from the wire?

1. $B = \dfrac{\mu_0 I}{2\pi r}$
2. $B = \dfrac{\mu_0 I}{\pi r}$
3. $B = \dfrac{2\mu_0 I}{\pi r}$
4. None of the above.



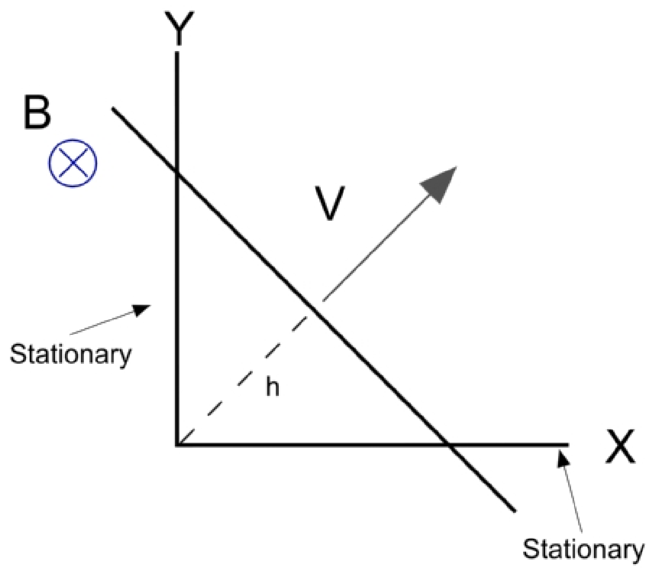
E.

A square coil rotates in the direction shown (see sketch) in a magnetic field directed to the right. At the time shown, the current in the square when looking ***down*** along its normal $\hat{\mathbf{n}}$ and the magnetic torque vector on the coil will be:

a) Current counterclockwise and torque ***out of the page*** along the rotation axis
b) Current counterclockwise and torque ***into the page along*** the rotation axis
c) Current clockwise and torque ***out of the page*** along the rotation axis
d) Current clockwise and torque ***into the page*** along the rotation axis

Problem 2: (25 points)

A long rigid straight conducting wire at a 45 degree angle as shown in the figure slides at a constant velocity $\vec{\mathbf{V}} = V_o(\hat{\mathbf{i}} + \hat{\mathbf{j}})/\sqrt{2}$ along an-infinite ***stationary*** L-shaped conducting wire. Note that $\left|\vec{\mathbf{V}}\right| = V_o$. Both wires lie in the *xy* plane, and together they form a closed triangular loop. The distance *h* from the origin to the center of the moving wire is given by $h = V_o t$ (see sketch). A uniform magnetic field $\vec{\mathbf{B}} = -B\,\hat{\mathbf{k}}$ points into the page. We consider only times $t > 0$. Both conducting wires have a resistance per unit length of α ohms per meter.



(a) Indicate on the sketch above with arrows the direction of the induced current in the loop. Give a Lenz's Law argument that justifies your answer (answers without justification, will not receive credit).

(b) What is the magnetic flux through the triangular loop at time *t* in terms of *t*, V_o , and *B*?

(c) Find the magnitude of the induced *emf* in terms of t, V_o, and B.

(d) Find the magnitude of the induced current in terms of V_o, α, and B.
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(e) Find the magnetic force acting on the moving wire in terms of the parameters given. Indicate magnitude and direction.

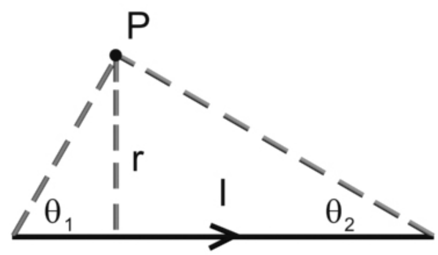
(f) Show that the rate at which the external agent moving the wire at constant velocity is doing work ($P_{external} = \vec{\mathbf{F}}_{external} \cdot \vec{\mathbf{V}}_o$) is equal to the rate at which energy is dissipated in the circuit due to Joule (Ohmic) heating.

Problem 3 **(25 points):**

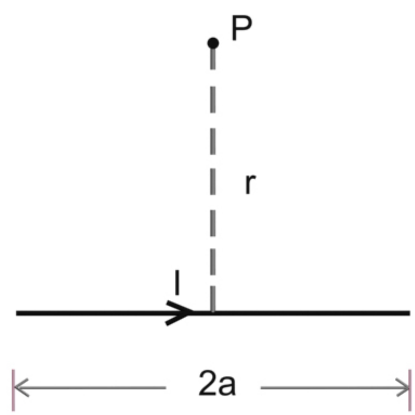
For a finite wire carrying a current *I* the contribution to the magnetic field at point *P* is

$$B(r) = \frac{\mu_0 I}{4\pi r}(\cos\theta_1 + \cos\theta_2)$$

where *B* is the magnitude of the magnetic field and θ_1 and θ_2 are the angles indicated in the figure:

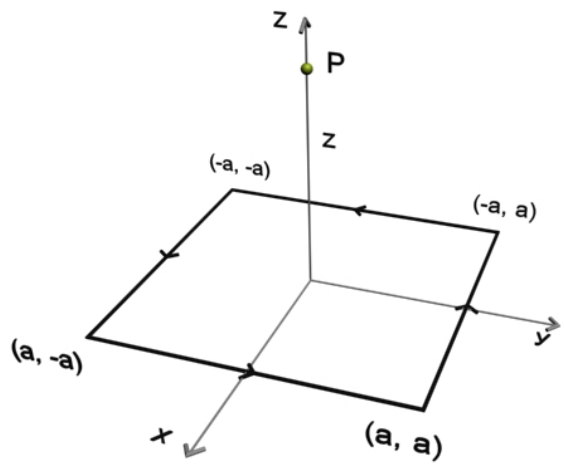


Consider a current *I* on a piece of wire of length 2*a*: (see figure below).



(a) Find the magnitude *B* of the magnetic field at point *P* located a distance *r* away from the wire and equidistant from the endpoints of the wire. On the above diagram indicate with an ⊗ or ⊙ whether the field points in or out of the page.

Consider a square loop of side 2*a* lying on the (*x, y*) plane as illustrated on the figure below (corners at (*x, y*) = $(\pm a, \pm a)$ and $(\pm a, \mp a)$). We are interested in the magnetic field at the point *P* on the *z* axis, a distance *z* from the origin.

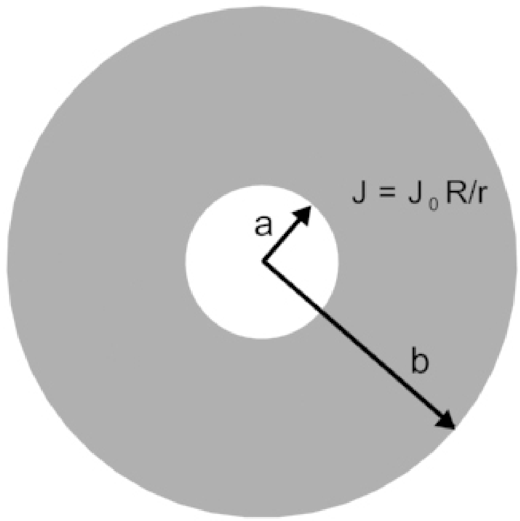


(b) What is the magnitude B_1 of the magnetic field at *P* contributed **separately** by each of the sides of the loop. Give your answer in terms of *a, z,* and other constants (μ_0, *I*, π, ...) [You may wish to use the result of (a).]

(c) Find the magnitude B_t of the **total** magnetic field at P. What direction does $\vec{\mathbf{B}}_t$ point?

(d) Calculate the leading value of the magnetic field B_t far away from the loop, namely, when $z >> a$. Write this magnetic field in terms of z and the dipole moment m of the current loop (as well as constants μ_0, π,…).

Problem 4 (25 points): A long cylindrical cable consists of a conducting cylindrical shell of inner radius *a* and outer radius *b*. The current density $\vec{\mathbf{J}}$ in the shell is out of the page (see sketch) and varies with radius as $J(r) = J_o \frac{R}{r}$ for $a < r < b$ and is zero outside of that range. Find the magnetic field in each of the following regions, indicating both magnitude and direction. Show your work and your Amperean loops.



(a) $r < a$

(b) $a < r < b$

(c) $r > b$. Give also $B(r = b)$, the value of the magnetic field at $r = b$.

(d) Plot your previous answers for the magnitude of B on the graph below.