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Chapter 7

FAULTS AND PROTECTION OF ELECTRIC ENERGY SYSTEMS

7.1 INTRODUCTION

A short-circuit fault takes place when two or more conductors come in contact with each other when normally they operate with a potential difference between them. The contact may be a physical metallic one, or it may occur through an arc. In the metal-to-metal contact case, the voltage between the two parts is reduced to zero. On the other hand, the voltage through an arc will be of a very small value. Short-circuit faults in three-phase systems are classified as:

1. Balanced or symmetrical three-phase faults.
2. Single line-to-ground faults.
3. Line-to-line faults.
4. Double line-to-ground faults.

Generator failure is caused by insulation breakdown between turns in the same slot or between the winding and the steel structure of the machine. The same can take place in transformers. The breakdown is due to insulation deterioration combined with switching and/or lightning overvoltages. Overhead lines are constructed of bare conductors. Wind, sleet, trees, cranes, kites, airplanes, birds, or damage to supporting structure are causes for accidental faults on overhead lines. Contamination of insulators and lightning overvoltages will in general result in short-circuit faults. Deterioration of insulation in underground cables results in short circuit faults. This is mainly attributed to aging combined with overloading. About 75 percent of the energy system’s faults are due to single-line-to-ground faults and result from insulator flashover during electrical storms. Only one in twenty faults is due to the balanced category.

A fault will cause currents of high value to flow through the network to the faulted point. The amount of current may be much greater than the designed thermal ability of the conductors in the power lines or machines feeding the fault. As a result, temperature rise may cause damage by annealing of conductors and insulation charring. In addition, the low voltage in the neighborhood of the fault will cause equipment malfunction.

Short-circuit and protection studies are an essential tool for the electric energy systems engineer. The task is to calculate the fault conditions and to provide protective equipment designed to isolate the faulted zone from the remainder of the system in the appropriate time. The least complex fault category computationally is the balanced fault. It is possible that a balanced fault could (in some locations) result in currents smaller than that due to some other type of fault. The interrupting capacity of breakers should be chosen to accommodate the largest of fault currents, and hence, care must be taken not to
base protection decisions on the results of a balanced three phase fault.

### 7.2 TRANSIENTS DURING A BALANCED FAULT

The value and severity of short-circuit current in the electric power system depends on the instant in the cycle at which the short circuit occurs. This can be verified using a simple model, consisting of a generator with series resistance \( R \) and inductance \( L \) as shown in Figure 7.1. The voltage of the generator is assumed to vary as

\[
e(t) = E_m \sin(\omega t + \alpha)
\]  

A dc term will in general exist when a balanced fault placed on the generator terminals at \( t = 0 \). The initial magnitude may be equal to the magnitude of the steady-state current term.

The worst possible case of transient current occurs for the value of short circuit placement corresponding to \( \alpha \) given by

\[
\tan \alpha = -\frac{R}{\omega L}.
\]

Here, the current magnitude will approach twice the steady-state maximum value immediately after the short circuit. The transient current is given in this case by the small \( t \) approximation

\[
i(t) = \frac{E_m}{Z} (1 - \cos \omega t)
\]  

It is clear that

---

**Figure 7.1** (a) Generator Model; (b) Voltage Waveform.
Figure 7.2 (a) Short-Circuit Current Wave Shape for $\tan \alpha = -(R/\omega L)$; (b) Short-Circuit Current Wave Shape for $\tan \alpha = (\omega L/R)$.

Figure 7.3 Symmetrical Short-Circuit Current and Reactances for a Synchronous Machine.

$$i_{\text{max}} = \frac{2E_m}{Z}$$

This waveform is shown in Figure 7.2(a).

For the case of short circuit application corresponding to

$$\tan \alpha = \frac{\omega L}{R}$$
we have

\[ i(t) = \frac{E_m}{Z} \sin \omega t \]  
(7.3)

This waveform is shown in Figure 7.2(b).

It is clear that the reactance of the machine appears to be time-varying, if we assume a fixed voltage source \( E \). For our power system purposes, we let the reactance vary in a stepwise fashion \( X_d^* \), \( X_d' \), and \( X_d \) as shown in Figure 7.3.

The current history \( i(t) \) can be approximated considering three time zones by three different expressions. The first is called the subtransient interval and lasts up to two cycles, the current is \( I'' \). This defines the direct-axis subtransient reactance:

\[ X_d^* = \frac{E}{I''} \]  
(7.4)

The second, denoted the transient interval, gives rise to

\[ X_d' = \frac{E}{I'} \]  
(7.5)

where \( I' \) is the transient current and \( X_d' \) is direct-axis transient reactance. The transient interval lasts for about 30 cycles.

The steady-state condition gives the direct-axis synchronous reactance:

\[ X_d = \frac{E}{I} \]  
(7.6)

Table 7.1 lists typical values of the reactances defined in Eqs. (7.4), (7.5), and (7.6). Note that the subtransient reactance can be as low as 7 percent of the synchronous reactance.

### 7.3 THE METHOD OF SYMMETRICAL COMPONENTS

The method of symmetrical components is used to transform an unbalanced three-phase system into three sets of balanced three-phase phasors. The basic idea of the transformations is simple. Given three voltage phasors \( V_A \), \( V_B \), and \( V_C \), it is possible to express each as the sum of three phasors as follows:

\[ V_A = V_{A+} + V_{A-} + V_{A0} \]  
(7.7)
Table 7.1

<table>
<thead>
<tr>
<th>Two-Pole Turbine Generator</th>
<th>Four-Pole Turbine Generator</th>
<th>Salient-Pole Machine with Dampers</th>
<th>Salient-Pole Generator without Dampers</th>
<th>Synchronous Condensers</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_d$</td>
<td>1.2</td>
<td>1.25</td>
<td>1.25</td>
<td>2.2</td>
</tr>
<tr>
<td>$X_d'$</td>
<td>0.15</td>
<td>0.30</td>
<td>0.30</td>
<td>0.48</td>
</tr>
<tr>
<td>$X_d''$</td>
<td>0.09</td>
<td>0.2</td>
<td>0.30</td>
<td>0.32</td>
</tr>
<tr>
<td>$X_0$</td>
<td>0.09</td>
<td>0.14</td>
<td>0.48</td>
<td>0.31</td>
</tr>
<tr>
<td>$X_0$</td>
<td>0.03</td>
<td>0.08</td>
<td>0.18</td>
<td>0.19</td>
</tr>
</tbody>
</table>

Figure 7.4 An Unbalanced Set of Voltage Phasors and a Possible Decomposition.

\[ V_B = V_{B+} + V_{B-} + V_{B0} \quad (7.8) \]

\[ V_C = V_{C+} + V_{C-} + V_{C0} \quad (7.9) \]

Figure 7.4 shows the phasors $V_A$, $V_B$, and $V_C$ as well as a particular possible choice of the decompositions.

Obviously there are many possible decompositions. For notational simplicity, we introduce the complex operator $\alpha$ defined by

\[ \alpha = e^{j120^\circ} \quad (7.10) \]

We require that the sequence voltages $V_{A+}$, $V_{B+}$, and $V_{C+}$ form a balanced positively rotating system. Thus the phasor magnitudes are equal, and the phasors are $120^\circ$ apart in a sequence $A-B-C$.

\[ V_{B+} = \alpha^2 V_{A+} \quad (7.11) \]
Similarly, we require that the sequence voltages $V_A^+$, $V_B^-$, and $V_C^-$ form a balanced negatively rotating system. This requires that the sequence is $C-B-A$

$$V_{B^-} = \alpha V_{A^-}$$  \hspace{1cm} (7.13)

$$V_{C^-} = \alpha^2 V_{A^-}$$  \hspace{1cm} (7.14)

The sequence voltages $V_{A^0}$, $V_{B^0}$, $V_{C^0}$ are required to be equal in magnitude and phase. Thus,

$$V_{B^0} = V_{A^0}$$  \hspace{1cm} (7.15)

$$V_{C^0} = V_{A^0}$$  \hspace{1cm} (7.16)

The original phasor voltages $V_A$, $V_B$, and $V_C$ are expressed in terms of the sequence voltages as

$$V_A = V_{A^+} + V_{A^-} + V_{A^0}$$  \hspace{1cm} (7.17)

$$V_B = \alpha^2 V_{A^+} + \alpha V_{A^-} + V_{A^0}$$  \hspace{1cm} (7.18)

$$V_C = \alpha V_{A^+} + \alpha^2 V_{A^-} + V_{A^0}$$  \hspace{1cm} (7.19)

The inverse relation giving the positive sequence voltage $V_{A^+}$, the negative sequence voltage $V_{A^-}$, and the zero sequence voltage $V_{A^0}$ is obtained by solving the above three simultaneous equations to give

$$V_{A^+} = \frac{1}{3} \left( V_A + \alpha V_B + \alpha^2 V_C \right)$$  \hspace{1cm} (7.20)

$$V_{A^-} = \frac{1}{3} \left( V_A + \alpha^2 V_B + \alpha V_C \right)$$  \hspace{1cm} (7.21)

$$V_{A^0} = \frac{1}{3} \left( V_A + V_B + V_C \right)$$  \hspace{1cm} (7.22)

Some of the properties of the operator $\alpha$ are as follows:
\[ \alpha^2 = \alpha^{-1} \]
\[ \alpha^3 = 1 \]
\[ 1 + \alpha + \alpha^2 = 0 \]

For clarity, we will drop the suffix A from the sequence voltage symbols, and we have

\[ V_A = V_+ + V_- + V_0 \quad (7.23) \]
\[ V_B = \alpha^2 V_+ + \alpha V_- + V_0 \quad (7.24) \]
\[ V_C = \alpha V_+ + \alpha^2 V_- + V_0 \quad (7.25) \]

\[ \alpha^2 = \alpha^{-1} \]
\[ \alpha^3 = 1 \]
\[ 1 + \alpha + \alpha^2 = 0 \]

Figure 7.5 (a) Positive Sequence Voltage Phasors; (b) Negative Sequence Voltage Phasors; and (c) Zero Sequence Voltage Phasors.
The ideas of symmetrical components apply to currents in the same manner.

We have the following two examples:

**Example 7.1**
The following currents were recorded under fault conditions in a three-phase system:

\[
I_A = 150\angle45^\circ \text{ A} \\
I_B = 250\angle150^\circ \text{ A} \\
I_C = 100\angle300^\circ \text{ A}
\]

Calculate the values of the positive, negative, and zero phase sequence components for each line.

**Solution**

\[
I_0 = \frac{1}{3}(I_A + I_B + I_C) \\
= \frac{1}{3}(106.04 + j106.07 + j106.07 - 216.51 + j125.00 + 50 - j86.6) \\
= 52.2\angle112.7^\circ
\]

\[
I_+ = \frac{1}{3}(I_A + \alpha I_B + \alpha^2 I_C) = \frac{1}{3}(150\angle45^\circ + 250\angle270^\circ + 100\angle180^\circ) \\
= 48.02\angle-87.6^\circ
\]

\[
I_- = \frac{1}{3}(I_A + \alpha^2 I_B + \alpha I_C) \\
= 163.21\angle40.45^\circ
\]

**Example 7.2**

Given that

\[
V_+ = \frac{1}{3}(V_A + \alpha V_B + \alpha^2 V_C) \quad (7.26)
\]

\[
V_- = \frac{1}{3}(V_A + \alpha^2 V_B + \alpha V_C) \quad (7.27)
\]

\[
V_0 = \frac{1}{3}(V_A + V_B + V_C) \quad (7.28)
\]
\[ V_0 = 100 \]
\[ V_A = 200\angle 60^\circ \]
\[ V_C = 100\angle 120^\circ \]

find the phase voltage \( V_A \), \( V_B \), and \( V_C \).

**Solution**

\[ V_A = V_+ + V_- + V_0 \]
\[ = 200\angle -120^\circ + 100\angle -60^\circ + 100 = 300\angle 60^\circ \]
\[ V_B = \alpha^2 V_+ + \alpha V_- + V_0 \]
\[ = (1\angle 240^\circ)(200\angle 60^\circ) + (1\angle 120^\circ)(100\angle 120^\circ) + 100 \]
\[ = 300\angle -60^\circ \]
\[ V_C = \alpha V_+ + \alpha^2 V_- + V_0 \]
\[ = (1\angle 120^\circ)(200\angle 60^\circ) + (1\angle 240^\circ)(100\angle 120^\circ) + 100 \]
\[ = 0 \]

**Power in Symmetrical Components**

The total power in a three-phase network is given in terms of phase variables by

\[ S = V_A^* I_A^* + V_B^* I_B^* + V_C^* I_C^* \] \hspace{1cm} (7.29)

where the asterisk denotes complex conjugation. We can show that the corresponding expression in terms of sequence variables is given by

\[ S = 3(V_+^* I_+^* + V_-^* I_-^* + V_0^* I_0^*) \] \hspace{1cm} (7.30)

The total power is three times the sum of powers in individual sequence networks.

### 7.4 SEQUENCE NETWORKS

**Positive Sequence Networks**

For a given power system the positive sequence network shows all the paths for the flow of positive sequence currents in the system. The one-line diagram of the system is converted to an impedance diagram that shows the equivalent circuit of each component under balanced operating conditions.
Each generator in the system is represented by a source voltage in series with the appropriate reactance and resistance. To simplify the calculations, all resistance and the magnetizing current for each transformer are neglected. For transmission lines, the line’s shunt capacitance and resistance are neglected. Motor loads, whether synchronous or induction, are included in the network as generated EMF’s in series with the appropriate reactance. Static loads are mostly neglected in fault studies.

**Negative Sequence Networks**

Three-phase generators and motors have only positive sequence-generated voltages. Thus, the negative sequence network model will not contain voltage sources associated with rotating machinery. Note that the negative sequence impedance will in general be different from the positive sequence values. For static devices such as transmission lines and transformers, the negative sequence impedances have the same values as the corresponding positive sequence impedances.

The current-limiting impedances between the generator’s neutral and ground will not appear in either the positive or negative sequence network. This arises simply because positive and negative sequence currents are balanced.

**Zero Sequence Networks**

The zero sequence network of a system depends on the nature of the connections of the three-phase windings for each of the system’s components.

**Delta-Connected Winding**

Zero sequence currents can exist in the phase windings of the delta connection. However, since we have the requirement

\[ I_{A0} = I_{B0} = I_{C0} = I_0 \]

we conclude that the line currents coming out of a delta winding are zero. For example,

\[ I_{AB} = I_{A0} - I_{B0} = 0 \]

This situation is shown in Figure 7.6.

The single-phase equivalent zero sequence network for a delta-connected load with zero sequence impedance \( Z_0 \) is shown in Figure 7.7.

**Wye-Connected Winding**

When a neutral return wire is present, zero sequence currents will pass both in the phase windings as well as on the lines. The neutral current \( I_n \) will be
This is shown in Figure 7.8(a). In the case of a system with no neutral return, \( I_N = 0 \) shows that no zero sequence currents can exist. This is shown in Figure 7.8(b). Zero sequence equivalents are shown in Figure 7.9.
Transformer’s Zero Sequence Equivalents

There are various possible combinations of the primary and secondary connections for three-phase transformers. These alter the corresponding zero sequence network.

**Delta-delta Bank**

Since for a delta circuit no return path for zero sequence current exists, no zero sequence current can flow into a delta-delta bank, although it can circulate within the delta windings. The equivalent circuit connections are shown in Figure 7.10.

**Wye-delta Bank, Ungrounded Wye**

For an ungrounded wye connection, no path exists for zero sequence current to the neutral. The equivalent circuit is shown in Figure 7.11.

**Wye-delta Bank, Grounded Wye**

Zero sequence currents will pass through the wye winding to ground. As a result, secondary zero sequence currents will circulate through the delta winding. No zero sequence current will exist on the lines of the secondary. The equivalent circuit is shown in Figure 7.12.
Figure 7.10 Zero Sequence Equivalent Circuits for a Three-Phase Transformer Bank Connected in delta-delta.

Figure 7.11 Zero Sequence Equivalent Circuits for a Three-Phase Transformer Bank Connected in Wye-delta.
Figure 7.12 Zero Sequence Equivalent Circuit for a Three-Phase Transformer Bank Connected in Wye-Delta Bank with Grounded Y.

Figure 7.13 Zero Sequence Equivalent Circuit for a Three-Phase Transformer Bank Connected in Wye-Wye with One Grounded Neutral.

**Wye-wye Bank, One Neutral Grounded**

With ungrounded wye, no zero sequence current can flow. No current in one winding means that no current exists in the other. Figure 7.13 illustrates the situation.
Figure 7.14 Zero Sequence Equivalent Circuit for a Three-Phase Transformer Bank Connected in Wye-Wye with Neutrals Grounded.

Wye-wye Bank, Both Neutrals Grounded

With both wyes grounded, zero sequence current can flow. The presence of the current in one winding means that secondary current exists in the other. Figure 7.14 illustrates the situation.

Sequence Impedances for Synchronous Machines

For a synchronous machine, sequence impedances are essentially reactive. The positive, negative, and zero sequence impedances have in general different values.

Positive Sequence Impedance

Depending on the time interval of interest, one of three reactances may be used:

1. For the subtransient interval, we use the subtransient reactance:
   \[ Z_s = jX_d^* \]

2. For the transient interval, we use the corresponding reactance:
   \[ Z_t = jX_d' \]

3. In the steady state, we have
   \[ Z_s = jX_d \]
Negative Sequence Impedance

The MMF produced by negative sequence armature current rotates in a direction opposite to the rotor and hence opposite to the dc field winding. Therefore the reactance of the machine will be different from that for the positively rotating sequence.

Zero Sequence Impedance

The zero sequence impedance of the synchronous machine is quite variable and depends on the nature of the stator windings. In general, these will be much smaller than the corresponding positive and negative sequence reactance.

Sequence Impedances for a Transmission Link

Consider a three-phase transmission link of impedance $Z_L$ per phase. The return (or neutral) impedance is $Z_N$. If the system voltages are unbalanced, we have a neutral current $I_N$. Thus,

$$I_N = I_A + I_B + I_C$$

The voltage drops $\Delta V_A$, $\Delta V_B$, and $\Delta V_C$ across the link are as shown below:

$$\Delta V_A = I_A Z_L + I_N Z_N$$
$$\Delta V_B = I_B Z_L + I_N Z_N$$
$$\Delta V_C = I_C Z_L + I_N Z_N$$

In terms of sequence voltages and currents, we have

$$\Delta V_+ = I_+ Z_L$$
$$\Delta V_- = I_- Z_L$$
$$\Delta V_0 = I_0 (Z_L + 3Z_N)$$

Therefore the sequence impedances are given by:

$$Z_0 = Z_L + 3Z_N$$
$$Z_- = Z_L$$
$$Z_+ = Z_L$$

The impedance of the neutral path entered into the zero sequence impedance in addition to the link’s impedance $Z_L$. However, for the positive and negative sequence impedances, only the link’s impedance appears.
Example 7.3
Draw the zero sequence network for the system shown in Figure 7.15.

Solution
The zero sequence network is shown in Figure 7.16.
Example 7.4
Obtain the sequence networks for the system shown in Figure 7.17. Assume the following data in p.u. on the same base:

**Generator G₁:**
- \( X_+ = 0.2 \) p.u.
- \( X_- = 0.12 \) p.u.
- \( X_0 = 0.06 \) p.u.

**Generator G₂:**
- \( X_+ = 0.33 \) p.u.
- \( X_- = 0.22 \) p.u.
- \( X_0 = 0.066 \) p.u.

**Transformer T₁:**
- \( X_+ = X_- = X_0 = 0.2 \) p.u.

**Transformer T₂:**
- \( X_+ = X_- = X_0 = 0.225 \) p.u.

**Transformer T₃:**
- \( X_+ = X_- = X_0 = 0.27 \) p.u.

**Transformer T₄:**
- \( X_+ = X_- = X_0 = 0.16 \) p.u.

**Line L₁:**
- \( X_+ = X_- = X_0 = 0.14 \) p.u.
- \( X_0 = 0.3 \) p.u.

**Line L₂:**
- \( X_+ = X_- = X_0 = 0.20 \) p.u.
- \( X_0 = 0.4 \) p.u.

**Line L₃:**
- \( X_+ = X_- = X_0 = 0.15 \) p.u.
- \( X_0 = 0.2 \) p.u.

**Load:**
- \( X_+ = X_- = 0.9 \) p.u.
- \( X_0 = 1.2 \) p.u.

Assume an unbalanced fault occurs at \( F \). Find the equivalent sequence networks for this condition.

![Figure 7.17 Network for Example 7.4.](image)

**Solution**
The positive sequence network is as shown in Figure 7.18(A). One step in the reduction can be made, the result of which is shown in Figure 7.18(B). To avoid tedious work we utilize Thévenin’s theorem to obtain the positive sequence network in reduced form. We assign currents \( I_1 \), \( I_2 \), and \( I_3 \) as shown in Figure 7.18(B) and proceed to solve for the open-circuit voltage between \( F_i \) and \( N_i \).
Consider loop A. We can write
\[1\angle 0 = j[0.2I_1 + 0.36(I_1 - I_3) + 0.9(I_1 + I_2)]\]

For loop B, we have
\[0 = j[0.565I_3 + 0.42(I_2 + I_3) - 0.36(I_1 - I_3)]\]

For loop C, we have
\[1\angle 0 = j[0.33I_2 + 0.42(I_2 + I_3) + 0.9(I_1 + I_2)]\]

The above three equations are rearranged to give
\[1\angle 0 = j(1.46I_1 + 0.9I_2 - 0.36I_3)\]
\[0 = 0.36I_1 - 0.42I_2 - 1.345I_3\]
\[1\angle 0 = j(0.9I_1 + 1.65I_2 + 0.42I_3)\]

Solving we obtain
\[ I_1 = -j0.4839 \]
\[ I_2 = -j0.3357 \]
\[ I_3 = -j0.0247 \]
Figure 7.19 (Cont.)
Figure 7.20 Positive Sequence Network Equivalent for Example 7.4.

Figure 7.21 Steps in Reduction of the Negative Sequence Network for Example 7.4.
Figure 7.21 (Cont.)
Figure 7.22 Steps in Reducing the Zero Sequence Network for Example 7.4.

As a result, we get
We now turn our attention to the Thévenin’s equivalent impedance, which is obtained by shorting out the sources and using network reduction. The steps are shown in Figure 7.19. As a result, we get

\[ Z_+ = j0.224 \]

The positive sequence equivalent is shown in Figure 7.20.

The negative sequence and zero sequence impedance networks and steps in their reduction are shown in Figure 7.21 and Figure 7.22. As a result, we get

\[ Z_- = j0.1864 \]
\[ Z_0 = j0.1315 \]

### 7.5 LINE-TO-GROUND FAULT

Assume that phase A is shorted to ground at the fault point \( F \) as shown in Figure 7.23. The phase \( B \) and \( C \) currents are assumed negligible, and we can thus write \( I_B = 0, I_C = 0 \). The sequence currents are obtained as:

\[ V_{F,N} = V_{TH} = 1 - j0.2I_1 - j0.16(I_1 - I_3) \]
\[ = 1 - (0.2)(0.4839) - (0.16)(0.4839 - 0.0247) \]
\[ = 0.82975 \]

\[ V_{F,N} = 0.82975 \]

\[ V_{nE} = 0.1864 \]

\[ V_{pC} = 0.1315 \]
With the generators normally producing balanced three-phase voltages, which are positive sequence only, we can write

\[ E_+ = E_A \]  
\[ E_- = 0 \]  
\[ E_0 = 0 \]

Let us assume that the sequence impedances to the fault are given by \( Z_+ \), \( Z_- \), \( Z_0 \). We can write the following expressions for sequence voltages at the fault:

\[ V_+ = E_+ - I_+ Z_+ \]  
\[ V_- = 0 - I_- Z_- \]  
\[ V_0 = 0 - I_0 Z_0 \]

The fact that phase A is shorted to ground is used. Thus,

\[ V_A = 0 \]

This leads to

\[ 0 = E_+ - I_0 (Z_+ + Z_- + Z_0) \]

or

\[ I_0 = \frac{E_+}{Z_+ + Z_- + Z_0} \]

The resulting equivalent circuit is shown in Figure 7.24.

We can now state the solution in terms of phase currents:

\[ I_A = \frac{3E_+}{Z_+ + Z_- + Z_0} \]  
\[ I_B = 0 \]  
\[ I_C = 0 \]
For phase voltages we have

\[ V_A = 0 \]
\[ V_B = \frac{E_B (1-\alpha) (Z_0 + (1+\alpha)Z_-)}{Z_0 + Z_- + Z_+} \]  \hspace{1cm} (7.40)
\[ V_C = \frac{E_C (1-\alpha) (1+\alpha) Z_0 + Z_-)}{Z_0 + Z_- + Z_+} \]

**Example 7.5**

Consider a system with sequence impedances given by \( Z_+ = j0.2577 \), \( Z_- = j0.2085 \), and \( Z_0 = j0.14 \); find the voltages and currents at the fault point for a single line-to-ground fault.

**Solution**

The sequence networks are connected in series for a single line-to-ground fault.

The sequence currents are given by

\[ I_* = I_- = I_0 = \frac{1}{j(0.2577 + 0.2085 + 0.14)} \]
\[ = 1.65 \angle -90^\circ \text{ p.u.} \]

Therefore,

\[ I_A = 3I_* = 4.95 \angle -90^\circ \text{ p.u.} \]
\[ I_B = I_C = 0 \]
The sequence voltages are as follows:

\[
V_s = E_s - I_s Z_s
\]
\[
= 1 \angle 0 - (1.65 \angle -90°)(0.2577 \angle 90°)
\]
\[
= 0.57 \text{ p.u.}
\]

\[
V_0 = -I_0 Z_0
\]
\[
= -(1.65 \angle -90°)(0.14 \angle 90°)
\]
\[
= -0.34 \text{ p.u.}
\]

The phase voltages are thus

\[
V_A = V_s + V_0 = 0
\]
\[
V_B = \alpha^2 V_s + \alpha V_0
\]
\[
= (1 \angle 240°)(0.57) + (1 \angle 120°)(-0.34) + (-0.23)
\]
\[
= 0.86 \angle -113.64° \text{ p.u.}
\]

\[
V_C = \alpha V_s + \alpha^2 V_0
\]
\[
= (1 \angle 120°)(0.57) + (1 \angle 240°)(-0.34) + (-0.23)
\]
\[
= 0.86 \angle 113.64° \text{ p.u.}
\]

### 7.6 DOUBLE LINE-TO-GROUND FAULT

We will consider a general fault condition. In this case we assume that phase B has fault impedance of \(Z_f\), phase C has a fault impedance of \(Z_f\), and the common line-to-ground fault impedance is \(Z_g\). This is shown in Figure 7.25.

The boundary conditions are as follows:

\[
I_A = 0
\]
\[
V_{Bn} = I_B (Z_f + Z_g) + I_C Z_g
\]
\[
V_{Cn} = I_B Z_g + (Z_f + Z_g) I_C
\]

We can demonstrate that

\[
E_s - I_s (Z_s + Z_f) = -I_0 (Z_s + Z_f)
\]
\[
= -I_0 (Z_s + Z_f + 3Z_g)
\]

\(7.41\)
The equivalent circuit is shown in Figure 7.26. It is clear from Eq. (7.41) that the sequence networks are connected in parallel. From the equivalent circuit we can obtain the positive, negative, and zero sequence currents easily.

**Example 7.6**
For the system of Example 7.5 find the voltages and currents at the fault point for a double line-to-ground fault. Assume
\[ Z_f = j0.05 \text{ p.u.} \]
\[ Z_s = j0.033 \text{ p.u.} \]

**Solution**

The sequence network connection is as shown in Figure 7.27. Steps of the network reduction are also shown. From the figure, sequence currents are as follows:

\[
I_+ = \frac{1 \angle 0}{0.45 \angle 90^\circ} = 2.24 \angle -90^\circ
\]
\[
I_- = -I_+ \frac{0.29}{0.29 + 0.2585} = -1.18 \angle -90^\circ
\]
\[
I_0 = -1.06 \angle -90^\circ
\]

The sequence voltages are calculated as follows.

\[
V_+ = E_+ - I_+ Z_+
\]
\[
= 1 \angle 0 - \left(2.24 \angle -90^\circ\right)0.26 \angle -90^\circ
\]
\[
= 0.42
\]
\[
V_- = -I_- Z_-
\]
\[
= +(1.18)(0.2085) = 0.25
\]
\[
V_0 = -I_0 Z_0
\]
\[
= (1.06)(0.14) = 0.15
\]

The phase currents are obtained as

\[
I_A = 0
\]
\[
I_B = \alpha^2 I_+ + \alpha I_- + I_0
\]
\[
= (1 \angle 240)(2.24 \angle -90^\circ) + (1 \angle 120)(-1.18 \angle -90^\circ)
\]
\[
+ \left(-1.06 \angle -90^\circ\right)
\]
\[
= 3.36 \angle 151.77^\circ
\]
\[
I_C = \alpha I_+ + \alpha^2 I_- + I_0
\]
\[
= (1 \angle 120)(2.24 \angle -90^\circ) + (1 \angle 240)(-1.18 \angle -90^\circ)
\]
\[
+ \left(-1.06 \angle -90^\circ\right)
\]
\[
= 3.36 \angle 28.23^\circ
\]
The phase voltages are found as

\[ V_A = V_+ + V_- + V_0 \]
\[ = 0.42 + 0.25 + 0.15 \]
\[ = 0.82 \]

\[ V_B = \alpha^2 V_+ + \alpha V_- + V_0 \]
\[ = \left( \frac{240^\circ}{2} \right) (0.42) + \left( \frac{120^\circ}{2} \right) (0.25) + (0.15) \]
\[ = 0.24 \angle -141.49^\circ \]

\[ V_C = \alpha V_+ + \alpha^2 V_- + V_0 \]
\[ = \left( \frac{120^\circ}{2} \right) (0.42) + \left( \frac{240^\circ}{2} \right) (0.25) + 0.15 \]
\[ = 0.24 \angle 141.49^\circ \]
7.7 **LINE-TO-LINE FAULT**

Let phase $A$ be the unfaulted phase. Figure 7.28 shows a three-phase system with a line-to-line short circuit between phases $B$ and $C$. The boundary conditions in this case are:

$$
I_A = 0 \\
I_B = -I_C \\
V_B - V_C = I_B Z_f
$$

The first two conditions yield:

$$
I_0 = 0 \\
I_+ = -I_- = \frac{1}{3} (\alpha - \alpha^2) I_B
$$

The voltage conditions give:

$$
V_+ - V_- = Z_f I_v \tag{7.42}
$$

The equivalent circuit will take on the form shown in Figure 7.29.

Note that the zero sequence network is not included since $I_0 = 0$.

**Example 7.7**

For the system of Example 7.5, find the voltages and currents at the fault point for a line-to-line fault through an impedance $Z_f = j0.05$ p.u.

**Solution**

The sequence network connection is as shown in Figure 7.30. From the diagram,

![Figure 7.28 Example of a Line-to-Line Fault.](image)
The phase currents are thus

\[ I_A = 0 \]
\[ I_B = -I_C \]
\[ = (\alpha^2 - \alpha)I \]
\[ = (1^2 - 1)(1.93\angle -90^\circ) \]
\[ = 3.34\angle -180^\circ \text{ p.u.} \]

The sequence voltages are

\[ 1/0 \]
\[ j0.26 \]
\[ j0.05 \]
\[ j0.2085 \]
The phase voltages are obtained as shown below:

\[ V_A = V_+ + V_- + V_0 = 0.9 \text{ p.u.} \]
\[ V_B = \alpha^2 V_+ + \alpha V_- + V_0 = (1240^\circ)(0.5) + (120^\circ)(0.4) = 0.46 \angle -169.11^\circ \]
\[ V_C = \alpha V_+ + \alpha^2 V_- + V_0 = (120^\circ)(0.5) + (240^\circ)(0.4) = 0.46 \angle 169.11^\circ \]

As a check, we calculate

\[ V_B - V_C = 0.17 \angle -90^\circ \]
\[ I_0Z_f = (3.34 \angle -180^\circ)(0.05 \angle 90^\circ) = 0.17 \angle -90^\circ \]

Hence,

\[ V_B - V_C = I_0Z_f \]

### 7.8 THE BALANCED THREE-PHASE FAULT

Let us now consider the situation with a balanced three-phase fault on phases A, B, and C, all through the same fault impedance \( Z_f \). This fault condition is shown in Figure 7.31. It is clear from inspection in Figure 7.31 that the phase voltage at the faults are given by

\[ V_A = I_1Z_f \quad (7.43) \]
Figure 7.31 A Balanced Three-Phase Fault.

![Figure 7.31 A Balanced Three-Phase Fault.](image)

\[ V_B = I_B Z_f \]  \hspace{1cm} (7.44)

\[ V_C = I_C Z_f \]  \hspace{1cm} (7.45)

We can show that

\[ I_+ = \frac{E}{Z_r + Z_f} \]  \hspace{1cm} (7.46)

\[ I_- = 0 \]  \hspace{1cm} (7.47)

\[ I_0 = 0 \]  \hspace{1cm} (7.48)

The implications of Eqs. (7.47) and (7.48) are obvious. No zero sequence nor negative sequence components of the current exist. Instead, only positive sequence quantities are obtained in the case of a balanced three-phase fault.

**Example 7.8**

For the system of Example 7.5, find the short-circuit currents at the fault point for a balanced three-phase fault through three impedances each having a value of \( Z_f = j0.05 \) p.u.

**Solution**

\[ I_{A_c} = I_s = \frac{1}{j(0.26 + 0.05)} = 3.23 \angle -90^\circ \]

### 7.9 SYSTEM PROTECTION, AN INTRODUCTION

The result of the preceding section provides a basis to determine the conditions that exist in the system under fault conditions. It is important to take
the necessary action to prevent the faults, and if they do occur, to minimize possible damage or possible power disruption. A protection system continuously monitors the power system to ensure maximum continuity of electrical supply with minimum damage to life, equipment, and property.

The following are consequences of faults:

1. Abnormally large currents will flow in parts of system with associated overheating of components.
2. System voltages will be off their normal acceptable levels, resulting in possible equipment damage.
3. Parts of the system will be caused to operate as unbalanced three-phase systems, which will mean improper operation of the equipment.

A number of requirements for protective systems provide the basis for design criteria.

1. **Reliability**: Provide both dependability (guaranteed correct operation in response to faults) and security (avoiding unnecessary operation). Reliability requires that relay systems perform correctly under adverse system and environmental conditions.
2. **Speed**: Relays should respond to abnormal conditions in the least possible time. This usually means that the operation time should not exceed three cycles on a 60-Hz base.
3. **Selectivity**: A relay system should provide maximum possible service continuity with minimum system disconnection.
4. **Simplicity and economy**: The requirements of simplicity and economy are common in any engineering design, and relay systems are no exception.

A protective system detects fault conditions by continuously monitoring variables such as current, voltage, power, frequency, and impedance. Measuring currents and voltages is performed by instrument transformers of the potential type (P.T.) or current type (C.T.). Instrument transformers feed the measured variables to the relay system, which in turn, upon detecting a fault, commands a circuit-interrupting device known as the circuit breaker (C.B.) to disconnect the faulted section of the system.

An electric power system is divided into protective zones for each apparatus in the system. The division is such that zones are given adequate protection while keeping service interruption to a minimum. A single-line diagram of a part of a power system with its zones of protection is given in Figure 7.32. It is to be noted that each zone is overlapped to avoid unprotected (blind) areas.
7.10 **PROTECTIVE RELAYS**

A relay is a device that opens and closes electrical contacts to cause the operation of other devices under electric control. The relay detects intolerable or undesirable conditions within an assigned area. The relay acts to operate the appropriate circuit breakers to disconnect the area affected to prevent damage to personnel and property.

We classify relays according to their function, that is, as measuring or on-off relays. The latter class is also known as all-or-nothing and includes relays such as time-lag relays, auxiliary relays, and tripping relays. Here the relay does not have a specified setting and is energized by a quantity that is

![Diagram of Typical Zones of Protection in Part of an Electric Power System](image-url)

**Figure 7.32** Typical Zones of Protection in Part of an Electric Power System.
either higher than that at which it operates or lower than that at which it resets.

The class of measuring relays includes a number of types with the common feature that they operate at a predetermined setting. Examples are as follows:

- **Current relays**: Operate at a predetermined threshold value of current. These include overcurrent and undercurrent relays.
- **Voltage relays**: Operate at a predetermined value of voltage. These include overvoltage and undervoltage relays.
- **Power relays**: Operate at a predetermined value of power. These include overpower and underpower relays.
- **Directional relays**: 
  (i) Alternating current: Operate according to the phase relationship between alternating quantities.
  (ii) Direct current: Operate according to the direction of the current and are usually of the permanent-magnetic, moving-coil pattern.
- **Differential relays**: Operate according to the scalar or vectorial difference between two quantities such as current, voltage, etc.
- **Distance relays**: Operate according to the “distance” between the relay’s current transformer and the fault. The “distance” is measured in terms of resistance, reactance, or impedance.

Relays are made up of one or more fault-detecting units along with the necessary auxiliary units. Basic units for relay systems can be classified as being electromechanical units, sequence networks, or solid-state units. The electromechanical types include those based on magnetic attraction, magnetic induction, D’Arsonval, and thermal principles. Static networks with three-phase inputs can provide a single-phase output proportional to positive, negative, or zero sequence quantities. These are used as fault sensors and are known as sequence filters. Solid-state relays use low power components, which are designed into logic units used in many relays.

**Electromechanical Relays**

We consider some electromechanical type relays such as the plunger unit, the clapper unit, the polar unit, and the induction disc types.

The **plunger type** has cylindrical coils with an external magnetic structure and a center plunger. The plunger moves upward to operate a set of contacts when the current or voltage applied to the coil exceeds a certain value. The moving force is proportional to the square of the current in the coil. These units are instantaneous since no delay is intentionally introduced.

**Clapper units** have a U-shaped magnetic frame with a movable armature across the open end. The armature is hinged at one side and spring-restrained at the other. When the electrical coil is energized, the armature moves toward the magnetic core, opening or closing a set of contacts with a
torque proportional to the square of the coil current. Clapper units are less accurate than plunger units and are primarily applied as auxiliary or "go/no go" units.

Polar units use direct current applied to a coil wound around the hinged armature in the center of the magnetic structure. A permanent magnet across the structure polarizes the armature-gap poles. Two nonmagnetic spacers, located at the rear of the magnetic frames, are bridged by two adjustable magnetic shunts. This arrangement enables the magnetic flux paths to be adjusted for pickup and contact action. With balanced air gaps the armature will float in the center with the coil deenergized. With the gaps unbalanced, polarization holds the armature against one pole with the coil deenergized. The coil is arranged so that its magnetic axis is in line with the armature and at a right angle to the permanent magnet axis. Current in the coil magnetizes the armature either north or south, increasing or decreasing any prior polarization of the armature. If the magnetic shunt adjustment normally makes the armature a north pole, it will move to the right. Direct current in the operating coil, which tends to make the contact end a south pole, will overcome this tendency, and the armature will move to the left to close the contacts.

Induction disc units employ the watt hour meter design and use the same operating principles. They operate by torque resulting from the interaction of fluxes produced by an electromagnet with those from induced currents in the plane of a rotatable aluminum disc. The unit shown in Figure 7.33 has three poles on one side of the disc and a common magnetic keeper on the opposite side. The main coil is on the center leg. Current (I) in the main coil produces flux (φ), which passes through the air gap and disc to the keeper. The flux φ is divided into φL through the left-hand leg and φR through the right-hand leg. A short-circuited lagging coil on the left leg causes φL to lag both φR and φ, producing a split-phase motor action. The flux φL induces a voltage V, and current I, flows, in phase, in the shorted lag coil. The flux φT is the total flux produced by the main coil current (I). The three fluxes cross the disc air gap and produce eddy currents in the disc. As a result, the eddy currents set up counter fluxes, and the interaction of the two sets of fluxes produces the torque that rotates the disc.

A spiral spring on the disc shaft conducts current to the moving contact. This spring, together with the shape of the disc and the design of electromagnet, provides a constant minimum operating current over the contact’s travel range. A permanent magnet with adjustable keeper (shunt) damps the disc, and the magnetic plugs in the electromagnet control the degree of saturation. The spring tension, the damping magnet, and the magnetic plugs allow separate and relatively independent adjustment of the unit’s inverse time overcurrent characteristics.

Solid-State Units

Solid-state, linear, and digital-integrated circuit logic units are combined in a variety of ways to provide modules for relays and relay systems.
Three major categories of circuits can be identified: (1) fault-sensing and data-processing logic units, (2) amplification logic units, and (3) auxiliary logic units.

Logic circuits in the fault-sensing and data-processing category employ comparison units to perform conventional fault-detection duties. Magnitude comparison logic units are used for overcurrent detection both of instantaneous and time overcurrent categories. For instantaneous overcurrent protection, a dc level detector, or a fixed reference magnitude comparator, is used. A variable reference magnitude comparator circuit is used to ground-distance protection. Phase-angle comparison logic circuits produce an output when the phase angle between two quantities is in the critical range. These circuits are useful for phase, distance, and directional relays.

7.11 TRANSFORMER PROTECTION

A number of fault conditions can arise within a power transformer. These include:

1. **Earth faults**: A fault on a transformer winding will result in currents that depend on the source, neutral grounding impedance, leakage reactance of the transformer, and the position of the fault in the windings. The winding connections also influence the magnitude of fault current. In the case of a Y-connected winding with neutral point connected to ground through an impedance \( Z_g \), the fault current depends on \( Z_g \) and is proportional to the distance of the fault from the neutral point. If the neutral is solidly grounded, the fault current is controlled by the leakage reactance, which depends on fault location. The reactance decreases as the fault becomes closer to the neutral point. As a result, the fault current is highest for a fault close to the neutral point. In the case
of a fault in a ∆-connected winding, the range of fault current is less than that for a Y-connected winding, with the actual value being controlled by the method of grounding used in the system. Phase fault currents may be low for a ∆-connected winding due to the high impedance to fault of the ∆ winding. This factor should be considered in designing the protection scheme for such a winding.

2. **Core faults** due to insulation breakdown can permit sufficient eddy-current to flow to cause overheating, which may reach a magnitude sufficient to damage the winding.

3. **Interturn faults** occur due to winding flashovers caused by line surges. A short circuit of a few turns of the winding will give rise to high currents in the short-circuited loops, but the terminal currents will be low.

4. **Phase-to-phase** faults are rare in occurrence but will result in substantial currents of magnitudes similar to earth faults’.

5. **Tank faults** resulting in loss of oil reduce winding insulation as well as producing abnormal temperature rises.

In addition to fault conditions within the transformer, abnormal conditions due to external factors result in stresses on the transformer. These conditions include: overloading, system faults, overvoltages, and underfrequency operation.

When a transformer is switched in at any point of the supply voltage wave, the peak values of the core flux wave will depend on the residual flux as well as on the time of switching. The peak value of the flux will be higher than the corresponding steady-state value and will be limited by core saturation. The magnetizing current necessary to produce the core flux can have a peak of eight to ten times the normal full-load peak and has no equivalent on the secondary side. This phenomenon is called **magnetizing inrush current** and appears as an internal fault. Maximum inrush occurs if the transformer is switched in when the supply voltage is zero. Realizing this, is important for the design of differential relays for transformer protection so that no tripping takes place due to the magnetizing inrush current. A number of schemes based on the harmonic properties of the inrush current are used to prevent tripping due to large inrush currents.

Overheating protection is provided for transformers by placing a thermal-sensing element in the transformer tank. Overcurrent relays are used as a backup protection with time delay higher than that for the main protection. Restricted earth fault protection is utilized for Y-connected windings. This scheme is shown in Figure 7.34. The sum of the phase currents is balanced against the neutral current, and hence the relay will not respond to faults outside the winding.
Differential protection is the main scheme used for transformers. The principle of a differential protection system is simple. Here the currents on each side of the protected apparatus for each phase are compared in a differential circuit. Any difference current will operate a relay. Figure 7.35 shows the relay circuit for one phase only. On normal operation, only the difference between the current transformer magnetizing currents $i_{m1}$ and $i_{m2}$ passes through the relay. This is due to the fact that with no faults within the protected apparatus, the currents entering and leaving are equal to $i$. If a fault occurs between the two sets of current transformers, one or more of the currents (in a three-phase system) on the left-hand side will suddenly increase, while that on the right-hand side may decrease or increase with a direction reversal. In both instances, the total fault current will flow through the relay, causing it to operate. In units where the neutral ends are inaccessible, differential relays are not used, but reverse power relays are employed instead.

A number of considerations should be dealt with in applying differential protection, including:
Figure 7.35 Basic Differential Connection.

1. Transformer ratio: The current transformers should have ratings to match the rated currents of the transformer winding to which they are applied.

2. Due to the $30^\circ$-phase change between Y- and $\Delta$-connected windings and the fact that zero sequence quantities on the Y side do not appear on the terminals of the $\Delta$ side, the current transformers should be connected in Y for a $\Delta$ winding and in $\Delta$ for a Y winding. Figure 7.36 shows the differential protection scheme applied to a $\Delta$/Y transformer. When current transformers are connected in $\Delta$, their secondary ratings must be reduced to $\left(1/\sqrt{3}\right)$ times the secondary rating of Y-connected transformers.

3. Allowance should be made for tap changing by providing restraining coils (bias). The bias should exceed the effect of the maximum ratio deviation.

Example 7.9
Consider a $\Delta$/Y-connected, 20-MVA, 33/11-kV transformer with differential protection applied, for the current transformer ratios shown in Figure 7.37. Calculate the relay currents on full load. Find the minimum relay current setting to allow 125 percent overload.

Solution
The primary line current is given by

$$I_p = \frac{20 \times 10^6}{\sqrt{3} \sqrt[3]{33 \times 10^3}} = 349.91 \text{ A}$$

The secondary line current is
Figure 7.36  Differential Protection of a Δ/Y Transformer.

Figure 7.37  Transformer for Example 7.9.

\[
I_s = \frac{20 \times 10^6}{\sqrt{3} \times 1 \times 10^3} = 1049.73 \text{ A}
\]

The C.T. current on the primary side is thus
\[ i_p = 349.91 \left(\frac{5}{300}\right) = 5.832 \, \text{A} \]

The C.T. current in the secondary side is

\[ i_s = 1049.73 \left(\frac{5}{2000}\right) \sqrt{3} = 4.545 \, \text{A} \]

Note that we multiply by \( \sqrt{3} \) to obtain the values on the line side of the \( \Delta \)-connected C.T.’s. The relay current on normal load is therefore

\[ i_r = i_p - i_s = 5.832 - 4.545 = 1.287 \, \text{A} \]

With 1.25 overload ratio, the relay setting should be

\[ I_r = (1.25)(1.287) = 1.61 \, \text{A} \]

**Buchholz Protection**

In addition to the above-mentioned protection schemes, it is common practice in transformer protection to employ gas-actuated relays for alarm and tripping. One such a relay is the Buchholz relay.

Faults within a transformer will result in heating and decomposing of the oil in the transformer tank. The decomposition produces gases such as hydrogen, carbon monoxide, and light hydrocarbons, which are released slowly for minor faults and rapidly for severe arcing faults. The relay is connected into the pipe leading to the conservator tank. As the gas accumulates, the oil level falls and a float \( F \) is lowered and operates a mercury switch to sound an alarm. Sampling the gas and performing a chemical analysis provide a means for classifying the type of fault. In the case of a winding fault, the arc generates gas at a high release rate that moves the vane \( V \) to cause tripping through contacts attached to the vane.

Buchholz protection provides an alarm for a number of fault conditions including:

1. Interturn faults or winding faults involving only lower power levels.
2. Core hot spots due to short circuits on the lamination insulation.
3. Faulty joints.
4. Core bolt insulation failure.
7.12 TRANSMISSION LINE PROTECTION

The excessive currents accompanying a fault, are the basis of overcurrent protection schemes. For transmission line protection in interconnected systems, it is necessary to provide the desired selectivity such that relay operation results in the least service interruption while isolating the fault. This is referred to as relay coordination. Many methods exist to achieve the desired selectivity. Time/current gradings are involved in three basic methods discussed below for radial or loop circuits where there are several line sections in series.

Three Methods of Relay Grading

A) Time Grading

Time grading ensures that the breaker nearest to the fault opens first, by choosing an appropriate time setting for each of the relays. The time settings increase as the relay gets closer to the source. A simple radial system shown in Figure 7.38 demonstrates this principle.

A protection unit comprising a definite time-delay overcurrent relay is placed at each of the points 2, 3, 4, and 5. The time-delay of the relay provides the means for selectivity. The relay at circuit breaker 2 is set at the shortest possible time necessary for the breaker to operate (typically 0.25 second). The relay setting at 3 is chosen here as 0.5 second, that of the relay at 4 at 1 second, and so on. In the event of a fault at F, the relay at 2 will operate and the fault will be isolated before the relays at 3, 4, and 5 have sufficient time to operate. The shortcoming of the method is that the longest fault-clearing time is associated with the sections closest to the source where the faults are most severe.

B) Current Grading

Fault currents are higher the closer the fault is to the source and this is utilized in the current-grading method. Relays are set to operate at a suitably graded current setting that decreases as the distance from the source is increased. Figure 7.39 shows an example of a radial system with current grading. The advantages and disadvantages of current grading are best illustrated by way of examples.

C) Inverse-Time Overcurrent Relaying

The inverse-time overcurrent relay method evolved because of the limitations imposed by the use of either current or time alone. With this method, the time of operation is inversely proportional to the fault current level, and the actual characteristics are a function of both time and current settings. Figure 7.40 shows some typical inverse-time relay characteristics. Relay type CO-7 is in common use. Figure 7.41 shows a radial system with time-graded inverse...
relays applied at breakers 1, 2, and 3.

For faults close to the relaying points, the inverse-time overcurrent method can achieve appreciable reductions in fault-clearing times.

The operating time of the time-overcurrent relay varies with the current magnitude. There are two settings for this type of relay:

1. *Pickup current* is determined by adjusted current coil taps or current tap settings (C.T.S.). The pickup current is the current that causes the relay to operate and close the contacts.
2. *Time dial* refers to the reset position of the moving contact, and it
varies the time of operation at a given tap setting and current magnitude.

The time characteristics are plotted in terms of time versus multiples of current tap (pickup) settings, for a given time dial position. There are five different curve shapes referred to by the manufacturer:

- CO-11: Extreme inverse
- CO-9: Very inverse
- CO-8: Inverse
- CO-7: Moderately inverse
- CO-6: Definite minimum

These shapes are given in Figure 7.40.

**Figure 7.40** Comparison of CO Curve Shapes.

**Example 7.10**
Consider the 11-kV radial system shown in Figure 7.42. Assume that all loads have the same power factor. Determine relay settings to protect the system assuming relay type CO-7 (with characteristics shown in Figure 7.43) is used.

**Solution**
The load currents are calculated as follows:
The normal currents through the sections are calculated as

\[
I_1 = \frac{4 \times 10^6}{\sqrt{3} (1 \times 10^3)} = 209.95 \text{ A}
\]

\[
I_2 = \frac{2.5 \times 10^6}{\sqrt{3} (1 \times 10^3)} = 131.22 \text{ A}
\]

\[
I_3 = \frac{6.75 \times 10^6}{\sqrt{3} (1 \times 10^3)} = 354.28 \text{ A}
\]

The normal currents through the sections are calculated as

\[
I_{21} = I_1 = 209.95 \text{ A}
\]

\[
I_{32} = I_{21} + I_2 = 341.16 \text{ A}
\]

\[
I_5 = I_{32} + I_3 = 695.44 \text{ A}
\]

With the current transformer ratios given, the normal relay currents are
We can now obtain the current tap settings (C.T.S.) or pickup current in such a manner that the relay does not trip under normal currents. For this type of relay, the current tap settings available are 4, 5, 6, 7, 8, 10, and 12 amperes. For position 1, the normal current in the relay is 5.25 A; we thus choose

\[
(C.T.S.)_1 = 6 \text{ A}
\]

For position 2, the normal relay current is 8.53 A, and we choose

\[
(C.T.S.)_2 = 10 \text{ A}
\]
Similarly for position 3,

\[(\text{C.T.S.})_3 = 10 \, \text{A}\]

Observe that we have chosen the nearest setting higher than the normal current.

The next task is to select the intentional delay indicated by the time dial setting (T.D.S.). We utilize the short-circuit currents calculated to coordinate the relays. The current in the relay at 1 on a short circuit at 1 is

\[i_{SC1} = \frac{2500}{200} = 62.5 \, \text{A}\]

Expressed as a multiple of the pickup or C.T.S. value, we have

\[\frac{i_{SC1}}{(\text{C.T.S.})_1} = \frac{62.5}{6} = 10.42\]

We choose the lowest T.D.S. for this relay for fastest action. Thus

\[(\text{T.D.S.})_1 = \frac{1}{2}\]

By reference to the relay characteristic, we get the operating time for relay 1 for a fault at 1 as

\[T_{11} = 0.15 \, \text{s}\]

To set the relay at 2 responding to a fault at 1, we allow 0.1 second for breaker operation and an error margin of 0.3 second in addition to \(T_{11}\). Thus,

\[T_{22} = T_{11} + 0.1 + 0.3 = 0.55 \, \text{s}\]

The short circuit for a fault at 1 as a multiple of the C.T.S. at 2 is

\[\frac{i_{SC1}}{(\text{C.T.S.})_2} = \frac{62.5}{10} = 6.25\]

From the characteristics for 0.55-second operating time and 6.25 ratio, we get

\[(\text{T.D.S.})_2 \equiv 2\]

The final steps involve setting the relay at 3. For a fault at bus 2, the
short-circuit current is 3000 A, for which relay 2 responds in a time $T_{22}$ obtained as follows:

$$\frac{i_{SC2}}{(C.T.S.)_2} = \frac{3000}{\left(\frac{200}{5}\right)10} = 7.5$$

For the (T.D.S.)$_2 = 2$, we get from the relay’s characteristic,

$$T_{22} = 0.50 \text{ s}$$

Thus allowing the same margin for relay 3 to respond to a fault at 2, as for relay 2 responding to a fault at 1, we have

$$T_{32} = T_{22} + 0.1 + 0.3$$

$$= 0.90 \text{ s}$$

The current in the relay expressed as a multiple of pickup is

$$\frac{i_{SC3}}{(C.T.S.)_3} = \frac{3000}{\left(\frac{400}{5}\right)10} = 3.75$$

Thus for $T_3 = 0.90$, and the above ratio, we get from the relay’s characteristic,

$$(T.D.S.)_3 \cong 2.5$$

We note here that our calculations did not account for load starting currents that can be as high as five to seven times rated values. In practice, this should be accounted for.

**Pilot-Wire Feeder Protection**

Graded overcurrent feeder protection has two disadvantages. First, the grading settings may lead to tripping times that are too long to prevent damage and service interruption. Second, satisfactory grading for complex networks is quite difficult to attain. This led to the concept of “unit protection” involving the measurement of fault currents at each end of a limited zone of the feeder and the transmission of information between the equipment at zone boundaries. The principle utilized here is the differential (often referred to as Merz-price) protection scheme. For short feeders, pilot-wire schemes are used to transmit the information. Pilot-wire differential systems of feeder protection are classified into three types: (1) the circulating-current systems, (2) the balanced-voltage systems, and (3) the phase-comparison (Casson-Last) system. All three systems depend on the fact that, capacitance current neglected, the instantaneous value of the current flowing into a healthy conductor at one end of the circuit is
equal to the instantaneous current flowing out of the conductor at the other end, so that the net instantaneous current flowing into or out of the conductor is zero if the conductor is healthy. If, on the other hand, the conductor is short-circuited to earth or to another conductor at some point, then the net current flowing into or out of the conductor is equal to the instantaneous value of the current flowing out of or into the conductor at the point of fault.

### 7.13 Impedance-Based Protection Principles

This section discusses the principles involved in protecting components such as transmission lines on the basis of measuring the input impedance of the component. We first discuss the idea of an \(X-R\) diagram which is an excellent graphical tool to demonstrate principles of impedance protection systems. The concept of relay compartors is then introduced. The specific parameter choices to allow for the creation of impedance relays based on either amplitude or phase comparisons are then discussed. The section concludes with a discussion of distance protection.

**A) The X-R Diagram**

Consider a transmission line with series impedance \(Z_L\) and negligible shunt admittance. At the receiving end, a load of impedance \(Z_R\) is assumed. The phasor diagram shown in Figure 7.44 is constructed with \(I\) taken as the reference. The phasor diagram represents the relation

\[ V_S = IZ_L + V_r \]

(7.49)

giving rise to the heavy-lines diagram rather than the usual one shown by the dashed line. On the diagram, \(\delta\) is the torque angle, which is the angle between \(V_s\) and \(V_r\).

If the phasor diagram, Eq. (7.49), is divided by the current \(I\), we obtain the impedance equation

\[ Z_s = Z_L + Z_r \]

(7.50)

![Figure 7.44 Voltage Phasor Diagram.](image)
An impedance diagram is shown in Figure 7.45. This is called the X-R diagram since the real axis represents a resistive component ($R$), and the imaginary axis corresponds to a reactive component ($X$). The angle $\delta$ appears on the impedance diagram as that between $Z_s$ and $Z_r$. The evaluation of $Z_r$ from complex power $S_R$ and voltage $V_r$ is straightforward.

**B) Relay Comparators**

Relay comparators can have any number of input signals. However, we focus our attention here on the two-input comparator shown schematically in Figure 7.46. The input to the two transformer circuits 1 and 2 includes the line voltage $V_L$ and current $I_L$. The output of transformer 1 is $V_1$, and that of transformer 2 is $V_2$. Both $V_1$ and $V_2$ are input to the comparator, which produces a trip (operate) signal whenever $|V_2| > |V_1|$ in an amplitude comparison mode.

We will start the analysis by assuming that the line voltage $V_L$ is the reference phasor and that the line current lags $V_L$ by and angle $\phi_L$. Thus,

$$V_L = |V_L| \angle 0$$
$$I_L = |I_L| \angle -\phi_L$$

The impedance $Z_L$ is thus
The transformers’ output voltages $V_1$ and $V_2$ are assumed to be linear combinations of the input quantities

$$V_1 = k_1 V_L + Z_I L$$

$$V_2 = k_2 V_L + Z_2 I_L$$

The impedances $Z_1$ and $Z_2$ are expressed in the polar form:

$$Z_1 = |Z_1| \angle \psi_1$$

$$Z_2 = |Z_2| \angle \psi_2$$

The comparator input voltages $V_1$ and $V_2$ are thus given by

$$V_1 = [I_L](k_1|Z_1| + |Z_2| \angle \psi_1 - \phi_L)$$

$$V_2 = [I_L](k_2|Z_1| + |Z_2| \angle \psi_2 - \phi_L)$$

C) Amplitude Comparison

The trip signal is produced for an amplitude comparator when

$$|V_2| \geq |V_1|$$

The operation condition is obtained as
\[
\left( k_1^2 - k_2^2 \right) |Z_L|^2 + 2|Z_L|[k_1|Z_1|\cos(\phi_1 - \phi_L) - k_2|Z_2|\cos(\phi_2 - \phi_L)] + (|Z_1|^2 - |Z_2|^2) \leq 0
\] (7.56)

This is the general equation for an amplitude comparison relay. The choices of \( k_1, k_2, Z_1, \) and \( Z_2 \) provide different relay characteristics.

**Ohm Relay**

The following parameter choice is made:

\[
k_1 = k \\
k_2 = -k \\
Z_1 = 0 \\
Z_2 = Z \\
\psi_1 = \psi_2 = \psi
\]

The relay threshold equation becomes

\[
R_L \cos \psi + X_L \sin \psi \leq \frac{|Z|}{2k}
\] (7.57)

This is a straight line in the \( X_L-R_L \) plane as shown in Figure 7.47. The shaded area is the restrain area; an operate signal is produced in the nonshaded area.

**Mho Relay**

The mho relay characteristic is obtained with the choice

\[
k_1 = -k \\
k_2 = 0 \\
Z_1 = Z_2 = Z \\
\psi_1 = \psi_2 = \psi
\]

\[
\left( R_L - \frac{|Z|}{k} \cos \psi \right)^2 + \left( X_L - \frac{|Z|}{k} \sin \psi \right)^2 \leq \frac{|Z|^2}{k^2}
\] (7.58)

The threshold condition with equality sign is a circle as show in Figure 7.48.

**Impedance Relay**

Here we set

\[
k_1 = -k \\
k_2 = 0 \\
Z_1 \neq Z_2
\]

The threshold equation is
The threshold condition is a circle with center at $|Z_1|/k \angle \psi$ and radius $|Z_2|/k$ as shown in Figure 7.49.

**Phase Comparison**

Let us now consider the comparator operating in the phase comparison mode. Assume that:

\[
\left( R_L - \frac{|Z_1| \cos \psi}{k} \right)^2 + \left( X_L - \frac{|Z_1| \sin \psi}{k} \right)^2 \leq \frac{|Z_2|^2}{k^2} \quad (7.59)
\]
Figure 7.49 Impedance Relay Characteristic.

\[ V_1 = |V_1| \angle \theta_1 \]
\[ V_2 = |V_2| \angle \theta_2 \]

Let the phase difference be defined as
\[ \theta = \theta_1 - \theta_2 \]

A criterion for operation of the ±90° phase comparator implies that
\[ \cos \theta \geq 0 \]

We can demonstrate that the general equation for the ±90° phase comparator is given by
\[ k_2 k_3 |Z_L|^2 + |Z_L| [k_1 |Z_1| \cos(\psi_1 - \phi_L) + k_2 |Z_1| \cos(\psi_1 - \phi_L)] + |Z_2| |Z_2| \cos(\psi_1 - \phi_L) \geq 0 \]  
(7.60)

By assigning values to the parameters \( k_2, k_3, Z_1, \) and \( Z_2, \) different relay characteristics such as the ohm and mho relays are obtained.

**D) Distance Protection**

Protection of lines and feeders based on comparison of the current values at both ends of the line can become uneconomical. Distance protection utilizes the current and voltage at the beginning of the line in a comparison scheme that essentially determines the fault position. Impedance measurement is performed using relay comparators. One input is proportional to the fault current and the other supplied by a current proportional to the fault loop voltage.
A plain impedance relay whose characteristic is that shown in Figure 7.49. It will thus respond to faults behind it (third quadrant) in the X-R diagram as well as in front of it. One way to prevent this is to add a separate directional relay that will restrain tripping for faults behind the protected zone. The reactance or mho relay with characteristics as shown in Figure 7.48 combines the distance-measuring ability and the directional property. The term mho is given to the relay where the circumference of the circle passes through the origin, and the term was originally derived from the fact that the mho characteristic (ohm spelled backward) is a straight line in the admittance plane.

Early applications of distance protection utilized relay operating times that were a function of the impedance for the fault. The nearer the fault, the shorter the operating time. This is shown in Figure 7.50. This has the same disadvantages as overcurrent protection discussed earlier. Present practice is to set the relay to operate simultaneously for faults that occur in the first 80 percent of the feeder length (known as the first zone). Faults beyond this point and up to a point midway along the next feeder are cleared by arranging for the zone setting of the relay to be extended from the first zone value to the second zone value after a time delay of about 0.5 to 1 second. The second zone for the first relay should never be less than 20 percent of the first feeder length. The zone setting extension is done by increasing the impedance in series with the relay voltage coil current. A third zone is provided (using a starting relay) extending from the middle of the second feeder into the third feeder up to 25 percent of the length with a further delay of 1 or 2 seconds. This provides backup protection as well. The time-distance characteristics for a three-feeder system are shown in Figure 7.51.

Distance relaying schemes employ several relay units that are arranged to give response characteristics such as that shown in Figure 7.52. A typical system comprises:

1. Two offset mho units (with three elements each). The first operates as earth-fault starting and third zone measuring relay, and the second operates as phase-fault starting and third zone measuring relay.
2. Two polarized mho units (with three elements each). The first unit acts as first and second zone earth-fault measuring relay, and the second unit acts as first and second zone phase-fault measuring relay.
3. Two time-delay relays for second and third zone time measurement.

The main difference between earth-fault and phase-fault relays is in the potential transformer (P.T.) and C.T. connections, which are designed to cause the relay to respond to the type of fault concerned.
E) Power Line Carrier Protection

The overhead transmission lines are used as pilot circuits in carrier-current protection systems. A carrier-frequency signal (30-200 kHz) is carried by two of the line conductors to provide communication means between ends of the line. The carrier signal is applied to the conductors via carrier coupling into units comprising inductance/capacitor circuits tuned to the carrier signal frequency to perform a number of functions. The carrier signals thus travel mainly into the power line and not into undesired parts of the system such as the bus bars. The communication equipment that operates at impedance levels of the order of 50-150 $\Omega$ is to be matched to the power line that typically has a characteristic impedance in the range of 240-500 $\Omega$.

Power line carrier systems are used for two purposes. The first involves measurements, and the second conveys signals from one end of the line to the other with the measurement being done at each end by relays. When the carrier channel is used for measurement, it is not practical to transmit amplitude measurements from one end to the other since signal attenuation beyond the control of the system takes place. As a result, the only feasible measurement carrier system compares the phase angle of a derived current at each end of the system in a manner similar to differential protection as discussed below.

Radio and microwave links have increasingly been applied in power
systems to provide communication channels for teleprotection as well as for supervisory control and data acquisition.

7.14 COMPUTER RELAYING

In the electric power industry computer-based systems have evolved to perform many complex tasks in energy control centers (treated in Chapter 8). Research efforts directed at the prospect of using digital computers to perform the tasks involved in power system protection date back to the mid-sixties and were motivated by the emergence of process-control computers. Computer relaying systems are now available. The availability of microprocessors used as a replacement for electromechanical and solid-state relays provides a number of advantages while meeting the basic protection philosophy requirement of decentralization.

There are many perceived benefits of a digital relaying system:

1. *Economics:* With the steady decrease in cost of digital hardware, coupled with the increase in cost of conventional relaying, it seems
reasonable to assume that computer relaying is an attractive alternative. Software development cost can be expected to be evened out by utilizing economies of scale in producing microprocessors dedicated to basic relaying tasks.

2. *Reliability:* A digital system is continuously active providing a high level of self-diagnosis to detect accidental failures within the digital relaying system.

3. *Flexibility:* Revisions or modifications made necessary by changing operational conditions can be accommodated by utilizing the programmability features of a digital system. This would lead to reduced inventories of parts for repair and maintenance purposes.

4. *System interaction:* The availability of digital hardware that monitors continuously the system performance at remote substations can enhance the level of information available to the control center. Postfault analysis of transient data can be performed on the basis of system variables monitored by the digital relay and recorded by the peripherals.

The main elements of a digital computer-based relay include:

1. Analog input subsystem
2. Digital input subsystem
3. Digital output subsystem
4. Relay logic and settings
5. Digital filters

The input signals to the relay are analog (continuous) and digital power system variables. The digital inputs are of the order of five to ten and include status changes (on-off) of contacts and changes in voltage levels in a circuit. The analog signals are the 60-Hz currents and voltages. The number of analog signals needed depends on the relay function but is in the range of 3 to 30 in all cases. The analog signals are scaled down (attenuated) to acceptable computer input levels (±10 volts maximum) and then converted to digital (discrete) form through analog/digital converters (ADC). These functions are performed in the “Analog Input Subsystem” block.

The digital output of the relay is available through the computer’s parallel output port. Five-to-ten digital outputs are sufficient for most applications. The analog signals are sampled at a rate between 240 Hz to about 2000 Hz. The sampled signals are entered into the scratch pad (random access memory (RAM)) and are stored in a secondary data file for historical recording. A digital filter removes noise effects from the sampled signals. The relay logic program determines the functional operations of the relay and uses the filtered sampled signals to arrive at a trip or no trip decision, which is then communicated to the system.

The heart of the relay logic program is a relaying algorithm that is
designed to perform the intended relay function such as overcurrent detection, differential protection, or distance protection, etc.

**PROBLEMS**

**Problem 7.1**
Consider the case of an open-line fault on phase $B$ of a three-phase system, such that

\[
\begin{align*}
I_A &= I \\
I_B &= 0 \\
I_C &= \alpha I
\end{align*}
\]

Find the sequence currents $I_+$, $I_-$, and $I_0$.

**Problem 7.2**
Consider the case of a three-phase system supplied by a two-phase source such that

\[
\begin{align*}
V_A &= V \\
V_B &= jV \\
V_C &= 0
\end{align*}
\]

Find the sequence voltages $V_+$, $V_-$, and $V_0$.

**Problem 7.3**
Calculate the phase currents and voltages for an unbalanced system with the following sequence values:

\[
\begin{align*}
I_a &= I_- = I_0 = -j1.0 \\
V_a &= 0.50 \\
V_- &= -0.30 \\
V_0 &= -0.20
\end{align*}
\]

**Problem 7.4**
Calculate the apparent power consumed in the system of Problem 7.3 using sequence quantities and phase quantities.

**Problem 7.5**
The zero and positive sequence components of an unbalanced set of voltages are

\[
\begin{align*}
V_+ &= 2 \\
V_0 &= 0.5 - j0.866
\end{align*}
\]

The phase $A$ voltage is
Obtain the negative sequence component and the $B$ and $C$ phase voltages.

**Problem 7.6**
Obtain the sequence networks for the system shown in Figure 7.53 in the case of a fault at $F$. Assume the following data in pu on the same base are given:

<table>
<thead>
<tr>
<th>Component</th>
<th>$X_a$</th>
<th>$X_b$</th>
<th>$X_c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Generator $G_1$:</td>
<td>0.2 p.u.</td>
<td>0.12 p.u.</td>
<td>0.06 p.u.</td>
</tr>
<tr>
<td>Generator $G_2$:</td>
<td>0.33 p.u.</td>
<td>0.22 p.u.</td>
<td>0.066 p.u.</td>
</tr>
<tr>
<td>Transformer $T_1$:</td>
<td>$X_a = X_b = X_c = 0.2$ p.u.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Transformer $T_2$:</td>
<td>$X_a = X_c = X_0 = 0.225$ p.u.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Transformer $T_3$:</td>
<td>$X_a = X_b = X_0 = 0.27$ p.u.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Transformer $T_4$:</td>
<td>$X_a = X_c = X_0 = 0.16$ p.u.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Line $L_1$:</td>
<td>$X_a = X_0 = 0.14$ p.u.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Line $L_2$:</td>
<td>$X_a = X_0 = 0.35$ p.u.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

![Figure 7.53 System for Problem 7.6.](image)

**Problem 7.7**
Assume an unbalanced fault occurs on the line bus of transformer $T_3$ in the system of Problem 7.6. Find the equivalent sequence networks for this condition.

**Problem 7.8**
Repeat Problem 7.7 for a fault on the generator bus of $G_2$.

**Problem 7.9**
Repeat Problem 7.7 for the fault in the middle of the line $L_1$. 

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Problem 7.10
Calculate the fault current for a single line-to-ground fault on phase A for a fault location as in Problem 7.7.

Problem 7.11
Repeat Problem 7.10 for a fault location in Problem 7.9.

Problem 7.12
Calculate the fault current in phase B for a double line-to-ground fault for a fault location as in Problem 7.7.

Problem 7.13
Repeat Problem 7.12 for a fault location as in Problem 7.8.

Problem 7.14
Repeat Problem 7.12 for a fault location in Problem 7.9.

Problem 7.15
Calculate the fault current in phase B for a line-to-line fault for a fault location as in Problem 7.7.

Problem 7.16
Repeat Problem 7.15 for a fault location as in Problem 7.8.

Problem 7.17
Repeat Problem 7.15 for a fault location as in Problem 7.9.

Problem 7.18
The following sequence voltages were recorded on an unbalanced fault:

\[ V_a = 0.5 \text{ p.u.} \]
\[ V_c = -0.4 \text{ p.u.} \]
\[ V_0 = -0.1 \text{ p.u.} \]

Given that the positive sequence fault current is \(-j1\), calculate the sequence impedances. Assume \(E = 1\).

Problem 7.19
The positive sequence current for a double line-to-ground fault in a system is \(-j1\) p.u., and the corresponding negative sequence current is \(0.333\) p.u. Given that the positive sequence impedance is 0.8 p.u., find the negative and zero sequence impedances.

Problem 7.20
The positive sequence current on a single line-to-ground fault on phase A at the load end of a radial transmission system is \(-j2\) p.u. For a double line-to-ground fault on phases B and C, the positive sequence current is \(-j3.57\) p.u., and for a
double-line fault between phases $B$ and $C$, its value is $-j2.67$. Assuming the sending-end voltage $E = 1.2$, find the sequence impedances for this system.

**Problem 7.21**
A turbine generator has the following sequence reactances:

$$
X_a = 0.1 \\
X_c = 0.13 \\
X_0 = 0.04
$$

Compare the fault currents for a three-phase fault and a single line-to-ground fault. Find the value of an inductive reactance to be inserted in the neutral connection to limit the current for a single line-to-ground fault to that for a three-phase fault.

**Problem 7.22**
A simultaneous fault occurs at the load end of a radial line. The fault consists of a line-to-ground fault on phase $A$ and a line-to-line fault on phases $B$ and $C$. The current in phase $A$ is $-j5$ p.u., whereas that in phase $B$ is $I_B = -3.46$ p.u. Given that $E = 1\angle 0^\circ$ and $Z_s = j0.25$, find $Z_a$ and $Z_0$.

**Problem 7.23**
Repeat Example 7.9, for a transformer rating of 12-MVA.

**Problem 7.24**
Consider the system of Example 7.10. Assume now that the load at the far end of the system is increased to

$$L_1 = 6 \text{ MVA}$$

Determine the relay settings to protect the system using relay type CO-7.

**Problem 7.25**
Consider the radial system of Example 7.10. It is required to construct the relay response time-distance characteristics on the basis of the design obtained as follows:

A. Assuming the line’s impedance is purely reactive, calculate the source reactance and the reactances between bus bars 3 and 2, and 2 and 1.
B. Find the current on a short circuit midway between buses 3 and 2 and between 2 and 1.
C. Calculate the relay response times for faults identified in Example 7.10 and part (B) above and sketch the relay response time-distance characteristics.
Problem 7.26
Consider a system with $|V_r| = 1 \text{ p.u.}$. Assume that the load is given by

$$S_r = 1 + j0.4 \text{ p.u.}$$

Find $Z_r$, $Z_s$, and the angle $\delta$ for this operating condition.

Problem 7.27
Assume that a line has an impedance $Z_L = 0.1 + j0.3 \text{ p.u.}$. The load is $S_r = 2 + j0.8 \text{ p.u.}$, $|V_r| = 1 \text{ p.u.}$. This line is to be provided with 80 percent distance protection using an ohm relay with $\psi = 45^\circ$. Find the relay’s impedance $Z$ assuming $k = 1$ and that magnitude comparison is used.

Problem 7.28
The line of Problem 7.27 is to be provided with 80 percent distance protection using either a resistance or a reactance ohm relay. Find the relay design parameters in each case, assuming that magnitude comparison is used.