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Chapter 5

ELECTRIC POWER TRANSMISSION

5.1 INTRODUCTION

The electric energy produced at generating stations is transported over high-voltage transmission lines to utilization points. The trend toward higher voltages is motivated by the increased line capacity while reducing line losses per unit of power transmitted. The reduction in losses is significant and is an important aspect of energy conservation. Better use of land is a benefit of the larger capacity.

This chapter develops a fundamental understanding of electric power transmission systems.

5.2 ELECTRIC TRANSMISSION LINE PARAMETERS

An electric transmission line is modeled using series resistance, series inductance, shunt capacitance, and shunt conductance. The line resistance and inductive reactance are important. For some studies it is possible to omit the shunt capacitance and conductance and thus simplify the equivalent circuit considerably.

We deal here with aspects of determining these parameters on the basis of line length, type of conductor used, and the spacing of the conductors as they are mounted on the supporting structure.

A wire or combination of wires not insulated from one another is called a conductor. A stranded conductor is composed of a group of wires, usually twisted or braided together. In a concentrically stranded conductor, each successive layer contains six more wires than the preceding one. There are two basic constructions: the one-wire core and the three-wire core.

Types of Conductors and Conductor Materials

Phase conductors in EHV-UHV transmission systems employ aluminum conductors and aluminum or steel conductors for overhead ground wires. Many types of cables are available. These include:

A. Aluminum Conductors

There are five designs:

1. Homogeneous designs: These are denoted as All-Aluminum-Conductors (AAC) or All-Aluminum-Alloy Conductors (AAAC).
2. Composite designs: These are essentially aluminum-
conductor-steel-reinforced conductors (ACSR) with steel core material.

3. Expanded ASCR: These use solid aluminum strands with a steel core. Expansion is by open helices of aluminum wire, flexible concentric tubes, or combinations of aluminum wires and fibrous ropes.


5. Aluminum-coated conductors.

B. Steel Conductors

Galvanized steel conductors with various thicknesses of zinc coatings are used.

**Line Resistance**

The resistance of the conductor is the most important cause of power loss in a power line. Direct-current resistance is given by the familiar formula:

\[ R_{dc} = \frac{\rho l}{A} \text{ ohms} \]

where

- \( \rho \) = resistivity of conductor
- \( l \) = length
- \( A \) = cross-sectional area

Any consistent set of units may be used in the calculation of resistance. In the SI system of units, \( \rho \) is expressed in ohm-meters, length in meters, and area in square meters. A system commonly used by power systems engineers expresses resistivity in ohms circular mils per foot, length in feet, and area in circular mils.

There are certain limitations in the use of this equation for calculating the resistance of transmission line conductors. The following factors need to be considered:

1. Effect of conductor stranding.
2. When ac flows in a conductor, the current is not distributed uniformly over the conductor cross-sectional area. This is called *skin effect* and is a result of the nonuniform flux distribution in the conductor. This increases the resistance of the conductor.
3. The resistance of magnetic conductors varies with current magnitude.
4. In a transmission line there is a nonuniformity of current distribution caused by a higher current density in the elements of adjacent conductors nearest each other than in the elements farther apart. The phenomenon is known as *proximity effect*. It is present
for three-phase as well as single-phase circuits. For the usual spacing of overhead lines at 60 Hz, the proximity effect is neglected.

5.3 **LINE INDUCTANCE**

The inductive reactance is by far the most dominating impedance element.

**Inductance of a Single-Phase Two-Wire Line**

The inductance of a simple two-wire line consisting of two solid cylindrical conductors of radii \( r_1 \) and \( r_2 \) shown in Figure 5.1 is considered first.

The total inductance of the circuit due to the current in conductor 1 only is given by:

\[
L_1 = (2 \times 10^7) \ln \left( \frac{D}{r_1'} \right)
\]  

Similarly, the inductance due to current in conductor 2 is

\[
L_2 = (2 \times 10^7) \ln \left( \frac{D}{r_2'} \right)
\]

Thus \( L_1 \) and \( L_2 \) are the phase inductances. For the complete circuit we have

\[
L_t = L_1 + L_2
\]

\[
L_t = (4 \times 10^7) \ln \left( \frac{D}{\sqrt{r_1' r_2'}} \right)
\]

where

\[
r_1' = r_1 e^{-\frac{1}{4}}
\]

\[
= 0.7788 r_i
\]

![Figure 5.1 Single-Phase Two-Wire Line Configuration.](image)
We compensate for the internal flux by using an adjusted value for the radius of the conductor. The quantity $r'$ is commonly referred to as the solid conductor’s geometric mean radius (GMR).

An inductive voltage drop approach can be used to get the same results

$$V_1 = j\omega(L_{11}I_1 + L_{12}I_2)$$ \hspace{1cm} (5.6)

$$V_2 = j\omega(L_{22}I_1 + L_{22}I_2)$$ \hspace{1cm} (5.7)

where $V_1$ and $V_2$ are the voltage drops per unit length along conductors 1 and 2 respectively. The self-inductances $L_{11}$ and $L_{22}$ correspond to conductor geometries:

$$L_{11} = (2 \times 10^{-7}) \ln \left( \frac{1}{r_1} \right)$$ \hspace{1cm} (5.8)

$$L_{22} = (2 \times 10^{-7}) \ln \left( \frac{1}{r_2} \right)$$ \hspace{1cm} (5.9)

The mutual inductance $L_{12}$ corresponds to the conductor separation $D$. Thus

$$L_{12} = (2 \times 10^{-7}) \ln \left( \frac{1}{D} \right)$$ \hspace{1cm} (5.10)

Now we have

$$I_2 = -I_1$$

The complete circuit’s voltage drop is

$$V_1 - V_2 = j\omega(L_{11} + L_{22} - 2L_{12})I_1$$ \hspace{1cm} (5.11)

In terms of the geometric configuration, we have

$$\Delta V = V_1 - V_2$$

$$\Delta V = j\omega(2 \times 10^{-7}) \left[ \ln \left( \frac{1}{r_1} \right) + \ln \left( \frac{1}{r_2} \right) - 2\ln \left( \frac{1}{D} \right) \right] I_1$$

$$= j\omega(4 \times 10^{-7}) \ln \left( \frac{D}{\sqrt{r_1r_2}} \right) I_1$$

Thus
\[ L_\gamma = (4 \times 10^{-7}) \ln \left( \frac{D}{\sqrt{r_1 r_2}} \right) \]  

(5.12)

where

\[ L_\gamma = L_{11} + L_{22} - 2L_{12} \]

We recognize this as the inductance of two series-connected magnetically coupled coils, each with self-inductance \( L_{11} \) and \( L_{22} \), respectively, and having a mutual inductance \( L_{12} \).

The phase inductance expressions given in Eqs. (5.1) and (5.2) can be obtained from the voltage drop equations as follows:

\[ V_1 = j \omega (2 \times 10^{-7}) \left[ I_1 \ln \left( \frac{1}{r_1} \right) + I_2 \ln \left( \frac{1}{D} \right) \right] \]

However, \( I_2 = -I_1 \)

Thus,

\[ V_1 = j \omega (2 \times 10^{-7}) \left[ I_1 \ln \left( \frac{D}{r_1} \right) \right] \]

In terms of phase inductance we have

\[ V_1 = j \omega L_\gamma I_1 \]

Thus for phase one,

\[ L_\gamma = (2 \times 10^{-7}) \ln \left( \frac{D}{r_1} \right) \text{ henries/meter} \]  

(5.13)

Similarly, for phase two,

\[ L_\gamma = (2 \times 10^{-7}) \ln \left( \frac{D}{r_2} \right) \text{ henries/meter} \]  

(5.14)

Normally, we have identical line conductors.
In North American practice, we deal with the inductive reactance of the line per phase per mile and use the logarithm to the base 10. Performing this conversion, we obtain

\[ X = k \log \frac{D}{r} \text{ ohms per conductor per mile} \]  

(5.15)

where

\[ k = 4.657 \times 10^{-3} f \]

(5.16)

assuming identical line conductors.

Expanding the logarithm in the expression of Eq. (5.15), we get

\[ X = k \log D + k \log \frac{1}{r} \]  

(5.17)

The first term is called \( X_d \) and the second is \( X_a \). Thus

\[ X_d = k \log D \text{ inductive reactance spacing factor in ohms per mile} \]  

(5.18)

\[ X_a = k \log \frac{1}{r} \text{ inductive reactance at 1-ft spacing in ohms per mile} \]  

(5.19)

Factors \( X_d \) and \( X_a \) may be obtained from tables available in many handbooks.

**Example 5.1**

Find the inductive reactance per mile per phase for a single-phase line with phase separation of 25 ft and conductor radius of 0.08 ft.

**Solution**

We first find \( r' \), as follows:

\[ r' = re^{-1/4} \]

\[ = (0.08)(0.7788) \]

\[ = 0.0623 \text{ ft} \]

We therefore calculate
The following MATLAB™ script implements Example 5.1 based on Eqs. (5.17) to (5.19)

```matlab
% Example 5-1
r=0.08
D=25
r_prime=0.7788*r
Xa=0.2794*(log10(1/(r_prime)))
Xb=0.2794*(log10 (D))
X=Xa+Xb
```

The answers obtained from MATLAB™ are as follows:

```matlab
EDU»
r = 0.0800
D = 25
r_prime = 0.0623
Xa = 0.3368
Xb = 0.3906
X = 0.7274
```

**Bundle Conductors**

At voltages above 230 kV (extra high voltage) and with circuits with only one conductor per phase, the corona effect becomes more excessive. Associated with this phenomenon is a power loss as well as interference with communication links. Corona is the direct result of high-voltage gradient at the conductor surface. The gradient can be reduced considerably by using more than one conductor per phase. The conductors are in close proximity compared with the spacing between phases. A line such as this is called a **bundle-conductor line**. The bundle consists of two or more conductors (subconductors) arranged on the perimeter of a circle called the **bundle circle** as shown in Figure 5.2. Another important advantage of bundling is the attendant reduction in line reactances, both series and shunt. The analysis of bundle-conductor lines is a specific case of the general multiconductor configuration problem.
Inductance of a Single-Phase Symmetrical Bundle-Conductor Line

Consider a symmetrical bundle with $N$ subconductors arranged in a circle of radius $A$. The angle between two subconductors is $2\pi/N$. The arrangement is shown in Figure 5.3.

We define the geometric mean distance (GMD) by

$$\text{GMD} = \left( \frac{D_{1(N+1)}}{D_{1(N+2)}} \right) \cdots \left( \frac{D_{1(2N)}}{D_{1(N+1)}} \right)^{1/N}$$  \hspace{1cm} (5.20)

Let us observe that practically the distances $D_{1(N+1)}$, $D_{1(N+2)}$, . . . , are all almost equal in value to the distance $D$ between the bundle centers. As a result,

$$\text{GMD} \equiv D$$  \hspace{1cm} (5.21)

Also, define the geometric mean radius as

$$\text{GMR} = \left[ Nr'(A)^{N-1} \right]^{1/N}$$  \hspace{1cm} (5.22)

The inductance is then obtained as
\[ L = (2 \times 10^{-7}) \ln \left( \frac{\text{GMD}}{\text{GMR}} \right) \] (5.23)

In many instances, the subconductor spacing \( S \) in the bundle circle is given. It is easy to find the radius \( A \) using the formula

\[ S = 2A \sin \left( \frac{\pi}{N} \right) \] (5.24)

which is a consequence of the geometry of the bundle as shown in Figure 5.4.

**Example 5.2**

Figure 5.5 shows a 1000-kv, single-phase, bundle-conductor line with eight subconductors per phase. The phase spacing is \( D_1 = 18 \) m, and the subconductor spacing is \( S = 50 \) cm. Each subconductor has a diameter of 5 cm. Calculate the line inductance.

**Solution**

We first evaluate the bundle radius \( A \). Thus,

\[ 0.5 = 2A \sin \left( \frac{\pi}{8} \right) \]
Therefore,

\[ A = 0.6533 \text{ m} \]

Assume that the following practical approximation holds:

\[ \text{GMD} = D_1 = 18 \text{ m} \]

The subconductor’s geometric mean radius is

\[
r_1' = 0.7788 \left( \frac{5 \times 10^{-2}}{2} \right)
= 1.947 \times 10^{-2} \text{ m}
\]

Thus we have

\[
L = (2 \times 10^{-7}) ln \left\{ \frac{\text{GMD}}{N_1'(A)^{N-1}} \right\}^{\frac{1}{N}}
= (2 \times 10^{-7}) ln \left\{ \frac{18}{(8)(1.947 \times 10^{-2})(0.6533)^7} \right\}^{\frac{1}{8}}
\]

The result of the above calculation is

\[ L = 6.99 \times 10^{-7} \text{ henries/meter} \]
The following MATLAB™ script implements Example 5.2 based on Eqs. (5.21) to (5.24)

```matlab
% Example 5-2
% N=8
S=0.5
d=0.05
r=d/2
r_prime=0.7788*r
GMD=18
A=(S/2)/sin(pi/N)
GMR=(N * r_prime * (A)^(N-1))^(1/N)
L=2*1e-7*log(GMD/GMR)
```

The answers obtained from MATLAB™ are as follows:

```
EDU»
N = 8
S = 0.5000
d = 0.0500
r = 0.0250
r_prime = 0.0195
GMD = 18
A = 0.6533
GMR = 0.5461
L = 6.9907e-007
```

**Inductance of a Balanced Three-Phase Single-Circuit Line**

We consider a three-phase line whose phase conductors have the general arrangement shown in Figure 5.6. We use the voltage drop per unit length concept. This is a consequence of Faraday’s law. In engineering practice we have a preference for this method. In our three-phase system, we can write

\[ V_i = j\omega(L_{11}I_1 + L_{12}I_2 + L_{13}I_3) \]
\[ V_2 = j\omega(L_{21}I_1 + L_{22}I_2 + L_{23}I_3) \]
\[ V_3 = j\omega(L_{31}I_1 + L_{32}I_2 + L_{33}I_3) \]

Here we generalize the expressions of Eqs. (5.8) and (5.10) to give

\[ L_{ii} = (2 \times 10^{-7}) \ln \left( \frac{1}{\eta_i} \right) \]  \hspace{1cm} (5.25)
We now substitute for the inductances in the voltage drops equations and use the condition of balanced operation to eliminate one current from each equation. The result is

\[ V'_1 = I_1 \ln \left( \frac{D_{13}}{r_1} \right) + I_2 \ln \left( \frac{D_{12}}{D_{13}} \right) \]
\[ V'_2 = I_1 \ln \left( \frac{D_{23}}{D_{12}} \right) + I_2 \ln \left( \frac{D_{23}}{r_2} \right) \]
\[ V'_3 = I_2 \ln \left( \frac{D_{13}}{D_{23}} \right) + I_3 \ln \left( \frac{D_{13}}{r_3} \right) \]

Here,

\[ V'_i = \frac{V_i}{j\omega(2\times10^{-7})} \]

We note that for this general case, the voltage drop in phase one, for example, depends on the current in phase two in addition to its dependence on \( I_1 \). Thus the voltage drops will not be a balanced system. This situation is undesirable.

Consider the case of equilaterally spaced conductors generally referred to as the \textit{delta} configuration; that is
\[ D_{12} = D_{13} = D_{23} = D \]
\[ r_1' = r_2' = r_3' = r' \]

The voltage drops will thus be given by

\[
\begin{align*}
V_1' &= I_1ln\left(\frac{D}{r'}\right) \\
V_2' &= I_2ln\left(\frac{D}{r'}\right) \\
V_3' &= I_3ln\left(\frac{D}{r'}\right)
\end{align*}
\] (5.28)

And in this case the voltage drops will form a balanced system.

Consider the so often called H-type configuration. The conductors are in one horizontal plane as shown in Figure 5.7. The distances between conductors are thus

\[
\begin{align*}
D_{12} &= D_{23} = D \\
D_{13} &= 2D
\end{align*}
\]

and the voltage drops are given by

\[
\begin{align*}
V_1' &= I_1ln\left(\frac{2D}{r'}\right) + I_2ln2 \\
V_2' &= I_2ln\left(\frac{D}{r'}\right) \\
V_3' &= I_3ln2 + I_3ln\left(\frac{2D}{r'}\right)
\end{align*}
\] (5.29)

We note that only conductor two has a voltage drop proportional to its current.

\[\text{Figure 5.7 H-Type Line.}\]
Transposition of Line Conductors

The equilateral triangular spacing configuration is not the only configuration commonly used in practice. Thus the need exists for equalizing the mutual inductances. One means for doing this is to construct transpositions or rotations of overhead line wires. A transposition is a physical rotation of the conductors, arranged so that each conductor is moved to occupy the next physical position in a regular sequence such as a-b-c, b-c-a, c-a-b, etc. Such a transposition arrangement is shown in Figure 5.8. If a section of line is divided into three segments of equal length separated by rotations, we say that the line is “completely transposed.”

Consider a completely transposed three-phase line. We can demonstrate that by completely transposing a line, the mutual inductance terms disappear, and the voltage drops are proportional to the current in each phase.

Define the geometric mean distance GMD as

\[ \text{GMD} = \left( D_{12} D_{13} D_{23} \right)^{1/3} \]  \hspace{1cm} (5.30)

and the geometric mean radius GMR as

\[ \text{GMR} = r' \]  \hspace{1cm} (5.31)

we attain


\[ L = (2 \times 10^{-7}) \ln \left( \frac{\text{GMD}}{\text{GMR}} \right) \text{henries/meter} \]  

(5.32)

Example 5.3
Calculate the inductance per phase of the three-phase solid conductor line shown in Figure 5.9. Assume that the conductor diameter is 5 cm and the phase separation \( D_l \) is 8 m. Assume that the line is transposed.

![Figure 5.9 A Three-Phase Line.](image)

Solution
The geometric mean distance is given by
\[
\text{GMD} = [D_l D_l (2D_l)]^{1/3} = 1.2599D_l = 10.08 \text{ m}
\]
The geometric mean radius is
\[
r' = \left( e^{-1/4} \right) \frac{5 \times 10^{-2}}{2} = 0.0195 \text{ m}
\]
Therefore,
\[
L = (2 \times 10^{-7}) \ln \left( \frac{10.08}{0.0195} \right) = 1.25 \times 10^{-6} \text{ henries/meter}
\]

Inductance of Multiconductor Three-Phase Systems

Consider a single-circuit, three-phase system with multiconductor-configured phase conductors as shown in Figure 5.10. Assume equal current.
distribution in the phase subconductors and complete transposition. We can show that the phase inductance for the system is the following expression:

\[
L = (2 \times 10^{-3}) \ln \left( \frac{GMD}{GMR} \right) \tag{5.33}
\]

In this case, the geometric mean distance is given by

\[
GMD = (D_{AB} D_{BC} D_{CA})^{1/3} \tag{5.34}
\]

where \(D_{AB}, D_{BC}, \) and \(D_{CA}\) are the distances between phase centers. The geometric mean radius (GMR) is obtained using the same expression as that for the single-phase system. Thus,

\[
GMR = \left[ \prod_{i=1}^{N} (D_{ni})^{1/N} \right] \tag{5.35}
\]

For the case of symmetrical bundle conductors, we have

\[
GMR = \left[ N r'(A)^{N-1} \right]^{1/N} \tag{5.36}
\]

The inductive reactance per mile per phase \(X_L\) in the case of a three-phase, bundle-conductor line can be obtained using

\[
X_L = X_a + X_d \tag{5.37}
\]

where as before for 60 Hz operation,
\[ X_a = 0.2794 \log \frac{1}{\text{GMR}} \quad (5.38) \]
\[ X_d = 0.2794 \log \text{GMD} \quad (5.39) \]

The GMD and GMR are defined by Eqs. (5.34) and (5.36).

**Example 5.4**

Consider a three-phase line with an eight subconductor-bundle delta arrangement with a 42 in. diameter. The subconductors are ACSR 84/19 (Chukar) with \( r' = 0.0534 \) ft. The horizontal phase separation is 75 ft, and the vertical separation is 60 ft. Calculate the inductive reactance of the line in ohms per mile per phase.

**Solution**

From the geometry of the phase arrangements, we have

\[
\tan \theta = \frac{36}{60} \\
\theta = 30.96^\circ \\
D_{AB} = \frac{60}{\cos 30.96^\circ} \\
= 69.97 \text{ ft}
\]

Thus,

\[
\text{GMD} = [(69.97)(69.97)(75)]^{1/3} = 71.577 \text{ ft}
\]

For Chukar we have \( r' = 0.0534 \) ft. The bundle particulars are \( N = 8 \) and \( A = (42/2) \) in. Therefore,

\[
\text{GMR} = \left[8(0.0534)\left(\frac{21}{12}\right)^7\right]^{1/8} \\
= 1.4672 \text{ ft}
\]

Thus,

\[
X_a = 0.2794 \log \frac{1}{1.4672} \\
= -0.0465 \\
X_d = 0.2794 \log 71.577 \\
= 0.518
\]
As a result,

\[ X_L = X_a + X_d = 0.4715 \text{ ohms per mile} \]

**Inductance of Three-Phase, Double-Circuit Lines**

A three-phase, double-circuit line is essentially two three-phase circuits connected in parallel. Normal practice calls for identical construction for the two circuits. If the two circuits are widely separated, then we can obtain the line reactance as simply half that of one single-circuit line. For the situation where the two circuits are on the same tower, the above approach may not produce results of sufficient accuracy. The error introduced is mainly due to neglecting the effect of mutual inductance between the two circuits. Here we give a simple but more accurate expression for calculating the reactance of double-circuit lines.

We consider a three-phase, double-circuit line with full line transposition such that in segment I, the relative phase positions are as shown in Figure 5.11.

The inductance per phase per unit length is given by

\[ L = \left(2 \times 10^{-7}\right) \ln \left(\frac{\text{GMD}}{\text{GMR}}\right) \]  \hspace{1cm} (5.40)

where the double-circuit geometric mean distance is given by

\[ \text{GMD} = \left(D_{AB_{m1}}D_{BC_{m1}}D_{AC_{m1}}\right)^{1/3} \]  \hspace{1cm} (5.41)

with mean distances defined by

\[ D_{AB_{m1}} = \left(D_{12}D_{12'}D_{12''}D_{12'''}\right)^{1/4} \]
\[ D_{BC_{m1}} = \left(D_{23}D_{23'}D_{23''}D_{23'''}\right)^{1/4} \]
\[ D_{AC_{m1}} = \left(D_{13}D_{13'}D_{13''}D_{13'''}\right)^{1/4} \]  \hspace{1cm} (5.42)

![Figure 5.11](image-url)  
**Figure 5.11** Double-Circuit Conductors' Relative Positions in Segment I of Transposition.
where subscript eq. refers to equivalent spacing. The GMR

\[
GMR = \left[ \frac{(GMR_A)(GMR_B)(GMR_C)}{3} \right]^{1/3}
\]  

(5.43)

with phase GMR’s defined by

\[
\begin{align*}
GMR_A &= \left[ r'(D_{11}) \right]^{1/2} \\
GMR_B &= \left[ r'(D_{22}) \right]^{1/2} \\
GMR_C &= \left[ r'(D_{33}) \right]^{1/2}
\end{align*}
\]  

(5.44)

We see from the above result that the same methodology adopted for the single-circuit case can be utilized for the double-circuit case.

**Example 5.5**

Calculate the inductance per phase for the three-phase, double-circuit line whose phase conductors have a GMR of 0.06 ft, with the horizontal conductor configuration as shown in Figure 5.12.

![Figure 5.12](image)

*Figure 5.12* Configuration for Example 5.5.

**Solution**

We use Eq. (5.42):

\[
\begin{align*}
D_{AB_{eq}} &= \left[ (25)(25)(50)(100) \right]^{1/4} \\
&= 42.04 \text{ ft} \\
D_{BC_{eq}} &= \left[ (25)(25)(50)(100) \right]^{1/4} \\
&= 42.04 \text{ ft} \\
D_{AC_{eq}} &= \left[ (50)(50)(125)(25) \right]^{1/4} \\
&= 52.87 \text{ ft}
\end{align*}
\]

As a result,

\[
GMD = \left[ (42.04)(42.04)(52.87) \right]^{1/3}
\]

\[= 45.381 \text{ ft}\]

The equivalent GMR is obtained using Eq. (5.44) as
\[
    r_{eq} = \left[0.06^3 \times (75)^3\right]^{1/6} = 2.121 \text{ ft}
\]

As a result,

\[
    L = (2 \times 10^{-7}) \ln \left(\frac{45.381}{2.121}\right) = 0.6126 \times 10^6 \text{ henries/meter}
\]

The following MATLAB™ script implements Example 5.5 based on Eqs. (5.40) to (5.44)

```matlab
% Example 5-5
% r_prime=0.06;
D_AAprime=75;
D_BBprime=75;
D_CCprime=75;
D_AB=25;
D_BC=D_AB;
D_CAprime=D_AB;
D_AprimeBprime=D_AB;
D_BprimeCprime=D_AB;
D_BCprime=D_AB+D_CAprime+D_AprimeBprime+D_BprimeCprime;
D_CBprime=D_CAprime+D_AprimeBprime;
D_ABprime=D_AB+D_BC+D_CAprime+D_AprimeBprime;
D_BAprime=D_BC+D_CAprime;
D_CA=D_AB+D_BC;
D_CprimeAprime=D_AprimeBprime+D_BprimeCprime;
D_ACprime=D_ABprime+D_BprimeCprime;
D_ABeq=(D_BC*D_BCprime*D_BprimeCprime*D_CBprime)^{(1/4)};
D_BCeq=(D_AprimeBprime*D_ABprime*D_AB*D_BAprime)^{(1/4)};
D_ACeq=(D_CA*D_CprimeAprime*D_CAprime*D_ACprime)^{(1/4)};
GMD=(D_ABeq*D_BCeq*D_ACeq)^{(1/3)};
% The equivalent GMR
r_eq=(r_prime^3*D_AAprime^3)^{(1/6)};
L=(2*10^-7)*log(GMD/r_eq);
```
The results of running the script are shown below:

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>EDU »</td>
<td></td>
</tr>
<tr>
<td>D_ABeq</td>
<td>42.0448</td>
</tr>
<tr>
<td>D_BCeq</td>
<td>42.0448</td>
</tr>
<tr>
<td>D_ACeq</td>
<td>52.8686</td>
</tr>
<tr>
<td>GMD</td>
<td>45.3810</td>
</tr>
<tr>
<td>r_eq</td>
<td>2.1213</td>
</tr>
<tr>
<td>L</td>
<td>6.1261e-07</td>
</tr>
</tbody>
</table>

5.4 **LINE CAPACITANCE**

The previous sections treated two line parameters that constitute the series impedance of the transmission line. The line inductance normally dominates the series resistance and determines the power transmission capacity of the line. There are two other line-parameters whose effects can be appreciable for high transmission voltages and line length. The line’s shunt admittance consists of the conductance \( g \) and the capacitive susceptance \( b \).

The conductance of a line is usually not a major factor since it is dominated by the capacitive susceptance \( b = \omega C \). The line capacitance is a leakage (or charging) path for the ac line currents.

The capacitance of a transmission line is the result of the potential differences between the conductors themselves as well as potential differences between the conductors and ground. Charges on conductors arise, and the capacitance is the charge per unit potential difference. Because we are dealing with alternating voltages, we would expect that the charges on the conductors are also alternating (i.e., time varying). The time variation of the charges results in what is called line-charging currents. In this section we treat line capacitance for a number of conductor configurations.

** Capacitance of Single-Phase Line**

Consider a single-phase, two-wire line of infinite length with conductor radii of \( r_1 \) and \( r_2 \) and separation \( D \) as shown in Figure 5.13. The potential at an arbitrary point \( P \) at distances \( r_a \) and \( r_b \) from \( A \) and \( B \), respectively, is given by

\[
V_p = \frac{q}{2\pi\varepsilon_0} \ln \left( \frac{r_b}{r_a} \right)
\]  

(5.45)

where \( q \) is the charge density in coulombs per unit length.

The potential \( V_A \) on the conductor \( A \) of radius \( r_1 \) is therefore obtained by setting \( r_a = r_1 \) and \( r_b = D \) to yield
Likewise for conductor \( B \) of radius \( r_2 \), we have

\[
V_B = \frac{q}{2\pi \varepsilon_0} \ln\left( \frac{r_2}{D} \right)
\]  

The potential difference between the two conductors is therefore

\[
V_{AB} = V_A - V_B = \frac{q}{\pi \varepsilon_0} \ln\left( \frac{D}{\sqrt{r_1 r_2}} \right)
\]  

The capacitance between the two conductors is defined as the charge on one conductor per unit of potential difference between the two conductors. As a result,

\[
C_{AB} = \frac{q}{V_{AB}} = \frac{\pi \varepsilon_0}{\ln\left( \frac{D}{\sqrt{r_1 r_2}} \right)} \text{ farads per meter}
\]  

If \( r_1 = r_2 = r \), we have

\[
C_{AB} = \frac{\pi \varepsilon_0}{\ln\left( \frac{D}{r} \right)}
\]  

Converting to microfarads \((\mu F)\) per mile and changing the base of the logarithmic term, we have
Equation (5.51) gives the line-to-line capacitance between the conductors. The capacitance to neutral for conductor $A$ is defined as

$$C_{AN} = \frac{q}{V_A} = \frac{2\pi\varepsilon_0}{\ln\left(\frac{D}{r_1}\right)}$$  \hfill (5.52)$$

Likewise, observing that the charge on conductor $B$ is $-q$, we have

$$C_{BN} = \frac{-q}{V_B} = \frac{2\pi\varepsilon_0}{\ln\left(\frac{D}{r_2}\right)}$$  \hfill (5.53)$$

For $r_1 = r_2$, we have

$$C_{AN} = C_{BN} = \frac{2\pi\varepsilon_0}{\ln\left(\frac{D}{r}\right)}$$  \hfill (5.54)$$

Observe that

$$C_{AN} = C_{BN} = 2C_{AB}$$  \hfill (5.55)$$

In terms of $\mu$F per mile, we have

$$C_{AN} = \frac{0.0388}{2\log\left(\frac{D}{r}\right)} \mu\text{F per mile}$$ \hfill (5.56)$$

The capacitive reactance $X_c$ is given by

$$X_c = \frac{1}{2\pi f C} = k' \log\left(\frac{D}{r}\right) \text{ ohms \cdot mile to neutral}$$  \hfill (5.57)$$

where

$$k' = \frac{4.1 \times 10^6}{f}$$  \hfill (5.58)$$
Expanding the logarithm, we have

$$X_c = k'\log D + k'\log \frac{1}{r}$$

(5.59)

The first term is called $X_{d'}$, the capacitive reactance spacing factor, and the second is called $X_{a'}$, the capacitive reactance at 1-ft spacing.

$$X_{d'} = k'\log D$$

(5.60)

$$X_{a'} = k'\log \frac{1}{r}$$

(5.61)

$$X_c = X_{d'} + X_{a'}$$

(5.62)

The last relationships are very similar to those for the inductance case. One difference that should be noted is that the conductor radius for the capacitance formula is the actual outside radius of the conductor and not the modified value $r'$.

**Example 5.6**

Find the capacitive reactance in ohms · mile per phase for a single-phase line with phase separation of 25 ft and conductor radius of 0.08 ft for 60-Hz operation.

**Solution**

Note that this line is the same as that of Example 5.1. We have for $f = 60$ Hz:

$$k' = \frac{4.1 \times 10^6}{f} = 0.06833 \times 10^6$$

We calculate

$$X_{d'} = k'\log 25$$

$$= 95.52 \times 10^1$$

$$X_{a'} = k'\log \frac{1}{0.08}$$

$$= 74.95 \times 10^1$$

As a result,

$$X_c = X_{d'} + X_{a'}$$

$$X_c = 170.47 \times 10^3$$ ohms · mile to neutral
The following MATLAB™ script implements Example 5.6 based on equations (5.60) to (5.62)

```matlab
% Example 5-6
% Data
f=60; % Hz
D=25; % phase separation (ft)
r=0.08; % conductor radius (ft)
% To calculate the capacitive reactance
kp=4.1*10^6/f;
Xdp=kp*log10(D)
Xap=kp*log10(1/r)
Xc=Xdp+Xap
```

The results of running the script are as shown below:

```
EDU»
Xdp =  9.5526e+004
Xap =  7.4956e+004
Xc =   1.7048e+005
```

**Including the Effect of Earth**

The effect of the presence of ground should be accounted for if the conductors are not high enough above ground. This can be done using the theory of image charges. These are imaginary charges of the same magnitude as the physical charges but of opposite sign and are situated below the ground at a distance equal to that between the physical charge and ground. The potential at ground due to the charge and its image is zero, which is consistent with the usual assumption that ground is a plane of zero potential.

**General Multiconductor Configurations**

Considering a system of \( n \) parallel and very long conductors with charges \( q_1, q_2, \ldots, q_n \), respectively, we can state that the potential at point \( P \) having distances \( r_1, r_2, \ldots, r_n \) to the conductor as shown in Figure 5.14 is given by

\[
V_p = \frac{q_1}{2\pi \varepsilon_0} \ln \left( \frac{1}{r_1} \right) + \frac{q_2}{2\pi \varepsilon_0} \ln \left( \frac{1}{r_2} \right) + \cdots + \frac{q_n}{2\pi \varepsilon_0} \ln \left( \frac{1}{r_n} \right) \quad (5.63)
\]

This is a simple extension of the two-conductor case.
If we consider the same $n$ parallel long conductors and wish to account for the presence of ground, we make use of the theory of images. As a result, we will have $n$ images charges $-q_1, -q_2, \ldots, -q_n$ situated below the ground at distance $q_1, q_2, \ldots, q_n$ from $P$. This is shown in Figure 5.15. The potential at $P$ is therefore

$$V_p = \frac{q_1}{2\pi \varepsilon_0} \ln \left( \frac{r_1}{\eta} \right) + \frac{q_2}{2\pi \varepsilon_0} \ln \left( \frac{r_2}{\eta} \right) + \cdots + \frac{q_n}{2\pi \varepsilon_0} \ln \left( \frac{r_n}{\eta} \right)$$

(5.64)

The use of this relationship in finding the capacitance for many systems will be treated next.

**Capacitance of a Single-Phase Line Considering the Effect of Ground**

Consider a single-phase line with conductors $A$ and $B$ as before. To account for ground effects, we introduce the image conductors $A'$ and $B'$. The situation is shown in Figure 5.16.
The voltage of phase $A$ is given according to Eq. (5.64) by

$$V_A = \frac{q}{2\pi\varepsilon_0} \ln \left( \frac{H}{r} \cdot \frac{D}{H_{AB}} \right) \quad (5.65)$$

The voltage of phase $B$ is

$$V_B = \frac{q}{2\pi\varepsilon_0} \ln \left( \frac{H_{AB}}{D} \cdot \frac{r}{H} \right) \quad (5.66)$$

The voltage difference is thus

$$V_{AB} = V_A - V_B$$

$$= \frac{q}{2\pi\varepsilon_0} \ln \left( \frac{H}{r} \cdot \frac{D}{H_{AB}} \right) \quad (5.67)$$

The capacitance between the two conductors is thus

$$C_{AB} = \frac{\varepsilon_0}{\ln \left( \frac{D}{r} \cdot \frac{H}{H_{AB}} \right)} \quad (5.68)$$

The capacitance to neutral is obtained using
\[ C_{AN} = \frac{q}{V_A} \]
\[ = \frac{2\pi \varepsilon_0}{\ln \left( \frac{D}{r} \cdot \frac{H}{H_{AB'}} \right)} \text{ farads per meter} \]  \hspace{1cm} (5.69)

Observe that again

\[ C_{AB} = \frac{C_{AN}}{2} \]

Let us examine the effect of including ground on the capacitance for a single-phase line in the following example.

**Example 5.7**
Find the capacitance to neutral for a single-phase line with phase separation of 20 ft and conductor radius of 0.075 ft. Assume the height of the conductor above ground is 80 ft.

**Solution**
We have

\[ D = 20 \text{ ft} \]
\[ r = 0.075 \text{ ft} \]
\[ H = 160 \text{ ft} \]

As a result,

\[ H_{AB'} = \sqrt{(160)^2 + (20)^2} = 161.2452 \text{ ft} \]

Therefore we have

\[ C_{AN} = \frac{2\pi \varepsilon_0}{\ln \left( \frac{20}{0.075} \cdot \frac{160}{161.2452} \right)} \]
\[ = \frac{2\pi \varepsilon_0}{5.578} \text{ farads per meter} \]

If we neglect earth effect, we have
The relative error involved if we neglect earth effect is:

\[
\frac{C_{AN_1} - C_{AN_2}}{C_{AN_1}} = 0.0014
\]

which is clearly less than 1 %.

**Capacitance of a Single-Circuit, Three-Phase Line**

We consider the case of a three-phase line with conductors not equilaterally spaced. We assume that the line is transposed and as a result can assume that the capacitance to neutral in each phase is equal to the average value. This approach provides us with results of sufficient accuracy for our purposes. This configuration is shown in Figure 5.17.

We use the three-phase balanced condition

\[
q_a + q_b + q_c = 0
\]

The average potential on phase A

\[
V_A = \frac{q_a}{2\pi \varepsilon_0} \ln \left( \frac{D_{12} D_{13} D_{23}}{r} \right)^{\frac{1}{3}}
\]

(5.70)

The capacitance to neutral is therefore given by

![Figure 5.17](image-url)  

*Figure 5.17* Three-Phase Line with General Spacing.
\[
C_{AN} = \frac{q_a}{V_A} = \frac{2\pi\varepsilon_0}{\ln\left(\frac{D_{eq}}{r}\right)}
\] (5.71)

where

\[
D_{eq} = \sqrt[3]{D_{12}D_{23}D_{13}}
\] (5.72)

Observe that \(D_{eq}\) is the same as the geometric mean distance obtained in the case of inductance. Moreover, we have the same expression for the capacitance as that for a single-phase line. Thus,

\[
C_{AN} = \frac{2\pi\varepsilon_0}{\ln\left(\frac{\text{GMD}}{r}\right)} \text{ farad per meter} \quad (5.73)
\]

If we account for the influence of earth, we come up with a slightly modified expression for the capacitance. Consider the same three-phase line with the attendant image line shown in Figure 5.18. The line is assumed to be transposed. As a result, the average phase A voltage will be given by

\[
V_A = \frac{q_a}{3(2\pi\varepsilon_0)} \ln\left[\frac{D_{12}D_{23}D_{13}\left(H_{12}H_{13}H_{23}\right)}{r^3(H_{12}H_{13}H_{23})}\right]
\] (5.74)

From the above,

Figure 5.18  Three-Phase Line with Ground Effect Included.
\[ C_{AN} = \frac{2\pi\varepsilon_0}{\ln \frac{D_{eq}}{r} \left( \frac{H_1 H_2 H_3}{H_{12} H_{13} H_{23}} \right)^{\frac{1}{3}}} \]

or

\[ C_{AN} = \frac{2\pi\varepsilon_0}{\ln \frac{D_{eq}}{r} + \ln \left( \frac{H_1 H_2 H_3}{H_{12} H_{13} H_{23}} \right)^{\frac{1}{3}}} \quad (5.75) \]

We define the mean distances

\[ H_s = (H_1 H_2 H_3)^{\frac{1}{3}} \quad (5.76) \]

\[ H_m = (H_{12} H_{23} H_{13})^{\frac{1}{3}} \quad (5.77) \]

Then the capacitance expression reduces to

\[ C_{AN} = \frac{2\pi\varepsilon_0}{\ln \left( \frac{D_{eq}}{r} \right) - \ln \left( \frac{H_m}{H_s} \right)} \quad (5.78) \]

We can thus conclude that including the effect of ground will give a higher value for the capacitance than that obtained by neglecting the ground effect.

**Figure 5.19** Conductor Layout for Example 5.8.

**Example 5.8**

Find the capacitance to neutral for the signal-circuit, three-phase, 345-kV line with conductors having an outside diameter of 1.063 in. with phase configuration as shown in Figure 5.19. Repeat including the effect of earth, assuming the height of the conductors is 50 ft.
Solution

\[
GMD = \left[ (23.5)(23.5)(47) \right]^{1/3} = 29.61 \text{ ft}
\]

\[
r = \frac{1.063}{(2)(12)} = 0.0443 \text{ ft}
\]

\[
C_{AN} = \frac{2\pi e_0}{\ln \left( \frac{GMD}{r} \right)} = 8.5404 \times 10^{-12} \text{ farads per meter}
\]

\[
H_1 = H_2 = H_3 = 2 \times 50 = 100 \text{ ft}
\]

\[
H_{12} = H_{23} = \sqrt{(23.5)^2 + (100)^2} = 102.72
\]

\[
H_{13} = \sqrt{(47)^2 + (100)^2} = 110.49
\]

\[
\ln \left( \frac{H_s}{H_m} \right) = \ln \left[ \frac{(100)(100)(100)}{(102.72)(102.72)(110.49)} \right]^{1/3} = -0.0512
\]

Thus,

\[
C_{AN} = \frac{1}{(18 \times 10^9)(6.505 - 0.0512)} = 8.6082 \times 10^{-12} \text{ farads per meter}
\]

The following MATLAB™ script implements Example 5.8.

```matlab
% Example 5-8
% data
D12=23.5; % ft
D23=23.5; % ft
D13=47; % ft
r=0.0443; % ft
eo=(1/(36*pi))*10^-9;
% To find the capacitance to neutral in farads/m
GMD=(D12*D23*D13)^(1/3)
CAN=(2*pi*eo)/(log(GMD/r))
% To calculate the capacitance to neutral,
% including the effect of earth
H1=2*50; % ft
H2=H1;
```

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MATLAB™ con’t.

The results of running the script are as shown below:

```matlab
H3=H1;
H12=(D12^2+H1^2).^5;
H23=H12;
H13=(D13^2+H3^2).^5;
Hs=(H1*H2*H3)^.(1/3);
Hm=(H12*H23*H13)^.(1/3);
CAN=(2*pi*eo)/(log(GMD/r)-log(Hm/Hs))
```

The average voltage of phase A can be shown to be given by

The calculation of capacitance of a double-circuit line can be quite involved if rigorous analysis is followed. In practice, however, sufficient accuracy is obtained if we assume that the charges are uniformly distributed and that the charge $q_a$ is divided equally between the two phase A conductors. We further assume that the line is transposed. As a result, capacitance formulae similar in nature to those for the single-circuit line emerge.

Consider a double-circuit line with phases, $A$, $B$, $C$, $A'$, $B'$, and $C'$ placed in positions 1, 2, 3, 1', 2', and 3', respectively, in segment $I$ of the transposition cycle. The situation is shown in Figure 5.20

The average voltage of phase $A$ can be shown to be given by

![Figure 5.20](image)

**Figure 5.20** Double-Circuit Line Conductor Configuration in Cycle Segment $I$ of Transposition.
\[ V_A = \frac{q_n}{12(2\pi\varepsilon_0)} \ln \left[ \left( \frac{D_{12} D_{12} D_{12} D_{12} D_{12} D_{12}}{r^6 D_{11}^2 D_{22}^2 D_{33}^2} \right) \left( D_{13} D_{13} D_{13} D_{13} D_{13} D_{13} \right) \right] \]  

(5.79)

As a result,  

\[ C_{AN} = \frac{2\pi\epsilon_0}{\ln \left( \frac{\text{GMD}}{\text{GMR}} + \alpha \right)} \]  

(5.80)

As before for the inductance case, we define  

\[ \text{GMD} = (D_{AB_{eq}} D_{BC_{eq}} D_{AC_{eq}})^{1/3} \]  

(5.81)

\[ D_{AB_{eq}} = (D_{12} D_{12} D_{12} D_{12} D_{12} D_{12})^{1/6} \]  

(5.82)

\[ D_{BC_{eq}} = (D_{23} D_{23} D_{23} D_{23} D_{23} D_{23})^{1/6} \]  

(5.83)

\[ D_{AC_{eq}} = (D_{13} D_{13} D_{13} D_{13} D_{13} D_{13})^{1/6} \]  

(5.84)

The GMR is given by  

\[ \text{GMR} = (r_A r_B r_C)^{1/3} \]  

(5.85)

with  

\[ r_A = (r D_{11})^{1/2} \]  

(5.86)

\[ r_B = (r D_{22})^{1/2} \]  

(5.87)

\[ r_C = (r D_{33})^{1/2} \]  

(5.88)

If we wish to include the effect of the earth in the calculation, a simple extension will do the job.

As a result, we have  

\[ C_{AN} = \frac{2\pi\epsilon_0}{\ln \left( \frac{\text{GMD}}{\text{GMR}} + \alpha \right)} \]  

(5.89)
where GMD and GMR are as given by Eqs. (5.81) and (5.85). Also, we defined

\[
\alpha = \ln\left(\frac{H_z}{H_m}\right) \tag{5.90}
\]

\[
H_s = \left(H_{s1} H_{s2} H_{s3}\right)^{1/3} \tag{5.91}
\]

with

\[
H_{s1} = \left(H_{12} H_{11} H_{11}^{2}\right)^{1/4} \tag{5.92}
\]

\[
H_{s2} = \left(H_{22} H_{22} H_{22}^{2}\right)^{1/4} \tag{5.93}
\]

\[
H_{s3} = \left(H_{33} H_{33} H_{33}^{2}\right)^{1/4} \tag{5.94}
\]

and

\[
H_m = \left(H_{m_{12}} H_{m_{12}} H_{m_{13}}\right)^{1/3} \tag{5.95}
\]

\[
H_{m_{12}} = \left(H_{12} H_{12} H_{12}^{2}\right)^{1/4} \tag{5.96}
\]
It should be evident by now that it is sufficient to consider a single-phase line to reach conclusions that can be readily extended to the three-phase case. We use this in the present discussion pertaining to bundle-conductor lines.

Consider a single-phase line with bundle conductor having $N$ subconductors on a circle of radius $A$. Each subconductor has a radius of $r$.

We have

$$C_{AN} = \frac{2\pi \varepsilon_0}{\ln \left( \frac{D}{D_{N-1}} \right)^{N}} \text{ farads per meter} \quad (5.99)$$

The extension of the above result to the three-phase case is clearly obtained by replacing $D$ by the GMD. Thus

$$C_{AN} = \frac{2\pi \varepsilon_0}{\ln \left( \frac{\text{GMD}}{D_{N-1}} \right)^{N}} \quad (5.100)$$

with

$$\text{GMD} = \left( D_{AB} D_{BC} D_{AC} \right)^{1/3} \quad (5.101)$$

The capacitive reactance in megaohms calculated for 60 Hz and 1 mile of line using the base 10 logarithm would be as follows:

$$X_c = 0.0683 \log \left( \frac{\text{GMD}}{D_{N-1}} \right)^{N} \quad (5.102)$$

$$X_c = X_d' + X_{d'} \quad (5.103)$$

This capacitive reactive reactance can be divided into two parts.
\[ X_a' = 0.0683\log\left(\frac{1}{\sqrt{N(A)^{N-1}N}}\right) \quad (5.104) \]

and

\[ X_d' = 0.0683\log(\text{GMD}) \quad (5.105) \]

If the bundle spacing \( S \) is specified rather than the radius \( A \) of the circle on which the conductors lie, then as before,

\[ A = \frac{S}{2\sin\left(\frac{\pi}{N}\right)} \quad \text{for } N > 1 \quad (5.106) \]

### 5.5 TWO-PORT NETWORKS

A network can have two terminals or more, but many important networks in electric energy systems are those with four terminals arranged in two pairs. A two-terminal pair network might contain a transmission line model or a transformer model, to name a few in our power system applications. The box is sometimes called a coupling network, or four-pole, or a two-terminal pair. The term two-port network is in common use. It is a common mistake to call it a four-terminal network. In fact, the two-port network is a restricted four-terminal network since we require that the current at one terminal of a pair must be equal and opposite to the current at the other terminal of the pair.

An important problem arises in the application of two-port network theory to electric energy systems, which is called the transmission problem. It is required to find voltage and current at one pair of terminals in terms of quantities at the other pair.

The transmission problem is handled by assuming a pair of equations of the form

\[ V_y = AV_x + BI_x \quad (5.107) \]

\[ I_y = CV_x + DI_x \quad (5.108) \]

to represent the two-port network. In matrix form, we therefore have

\[
\begin{bmatrix}
V_y \\
I_y
\end{bmatrix} =
\begin{bmatrix}
A & B \\
C & D
\end{bmatrix}
\begin{bmatrix}
V_x \\
I_x
\end{bmatrix}
\]

\[ (5.109) \]
For bilateral networks we have

\[ AD - BC = 1 \]  

(5.110)

Thus, there are but three independent parameters in the \( ABCD \) set as well.

Symmetry of a two-port network reduces the number of independent parameters to two. The network is *symmetrical* if it can be turned end for end in a system without altering the behavior of the rest of the system. An example is the transmission line, as will be seen later on. To satisfy this definition, a symmetrical network must have

\[ A = D \]  

(5.111)

We consider an important two-port network that plays a fundamental role in power system analysis – this is the symmetrical \( \pi \)-network. Figure 5.22 shows a symmetrical \( \pi \)-network. We can show that

\[ A = \left( 1 + \frac{Zy}{2} \right) \]  

(5.112)

\[ B = Z \]  

(5.113)

\[ C = Y \left( 1 + \frac{Zy}{4} \right) \]  

(5.114)

\[ D = A \]  

(5.115)

One of the most valued aspects of the \( ABCD \) parameters is that they are readily combined to find overall parameters when networks are connected in cascade. Figure 5.23 shows two cascaded two-part networks. We can write

Figure 5.22  A \( \pi \)-Network.
Figure 5.23 A Cascade of Two two-Port Networks.

\[
\begin{bmatrix}
V_s \\
I_s \\
V_M \\
I_M
\end{bmatrix} =
\begin{bmatrix}
A_1 & B_1 \\
C_1 & D_1
\end{bmatrix}
\begin{bmatrix}
V_M \\
I_M
\end{bmatrix},
\]

\[
\begin{bmatrix}
V_M \\
I_M
\end{bmatrix} =
\begin{bmatrix}
A_2 & B_2 \\
C_2 & D_2
\end{bmatrix}
\begin{bmatrix}
V_r \\
I_r
\end{bmatrix}
\]

From which, eliminating \((V_M, I_M)\), we obtain

\[
\begin{bmatrix}
V_s \\
I_s
\end{bmatrix} =
\begin{bmatrix}
A_1 & B_1 & A_2 & B_2 \\
C_1 & D_1 & C_2 & D_2
\end{bmatrix}
\begin{bmatrix}
V_r \\
I_r
\end{bmatrix}
\]

Thus the equivalent \(ABCD\) parameters of the cascade are

\[
A = A_1A_2 + B_1C_2 \tag{5.116}
\]

\[
B = A_1B_2 + B_1D_2 \tag{5.117}
\]

\[
C = C_1A_2 + D_1C_2 \tag{5.118}
\]

\[
D = C_1B_2 + D_1D_2 \tag{5.119}
\]

If three networks or more are cascaded, the equivalent \(ABCD\) parameters can be obtained most easily by matrix multiplications as was done above.

### 5.6 TRANSMISSION LINE MODELS

The line parameters discussed in the preceding sections were obtained on a per-phase, per unit length basis. We are interested in the performance of lines with arbitrary length, say \(l\). To be exact, one must take an infinite number of incremental lines, each with a differential length. Figure 5.24 shows the line with details of one incremental portion \((dx)\) at a distance \(x\) from the receiving end.

The assumptions used in subsequent analyses are:

1. The line is operating under sinusoidal, balanced, steady-state
Figure 5.24 Incremental Length of the Transmission Line.

2. The line is transposed.

With these assumptions, we analyze the line on a per phase basis. Application of Kirchhoff’s voltage and current relations yields

\[ \Delta V = I(x) \Delta x \]
\[ \Delta I = V(x) \Delta x \]

Let us introduce the propagation constant \( \nu \) defined as

\[ \nu = \sqrt{\frac{z}{y}} \] (5.120)

The series impedance per-unit length is defined by

\[ z = R + j\omega L \] (5.121)

The shunt admittance per-unit length is defined by

\[ y = G + j\omega C \] (5.122)

Let us introduce the propagation constant \( \nu \) defined as

\[ \nu = \sqrt{\frac{z}{y}} \] (5.120)

The series impedance per-unit length is defined by

\[ z = R + j\omega L \] (5.121)

The shunt admittance per-unit length is defined by

\[ y = G + j\omega C \] (5.122)

\( R \) and \( L \) are series resistance and inductance per unit length, and \( G \) and \( C \) are shunt conductance and capacitance to neutral per unit length.

In the limit, as \( \Delta x \to 0 \), we can show that

\[ \frac{d^2V}{dx^2} = \nu^2V \] (5.123)
\[ \frac{d^2I}{dx^2} = \nu^2I \] (5.124)

Equation (5.123) can be solved as an ordinary differential equation in
V. The solution turns out to be

\[ V(x) = A_1 \exp(ux) + A_2 \exp(-ux) \]  \hspace{1cm} (5.125)

Now taking the derivative of \( V \) with respect to \( x \) to obtain \( I(x) \) as

\[ I(x) = \frac{A_1 \exp(ux) - A_2 \exp(-ux)}{Z_c} \]  \hspace{1cm} (5.126)

Here we introduce

\[ Z_c = \sqrt{\frac{z}{y}} \]  \hspace{1cm} (5.127)

\( Z_c \) is the characteristic (wave) impedance of the line.

The constants \( A_1 \) and \( A_2 \) may be evaluated in terms of the initial conditions at \( x = 0 \) (the receiving end). Thus we have

\[ V(0) = A_1 + A_2 \]
\[ Z_c I(0) = A_1 - A_2 \]

from which we can write

\[ V(x) = \frac{1}{2} \left[ V(0) + Z_c I(0) \right] \exp(ux) + \left[ V(0) - Z_c I(0) \right] \exp(-ux) \]  \hspace{1cm} (5.128)

\[ I(x) = \frac{1}{2} \left[ I(0) + \frac{V(0)}{Z_c} \right] \exp(ux) + \left[ I(0) - \frac{V(0)}{Z_c} \right] \exp(-ux) \]  \hspace{1cm} (5.129)

Equations (5.128) and (5.129) can be used for calculating the voltage and current at any distance \( x \) from the receiving end along the line. A more convenient form of these equations is found by using hyperbolic functions.

We recall that

\[ \sinh \theta = \frac{\exp(\theta) - \exp(-\theta)}{2} \]
\[ \cosh \theta = \frac{\exp(\theta) + \exp(-\theta)}{2} \]

By rearranging Eqs. (5.128) and (5.129) and substituting the hyperbolic function for the exponential terms, a new set of equations is found. These are

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\[ V(x) = V(0) \cosh ux + Z_c I(0) \sinh ux \] (5.130)

and

\[ I(x) = I(0) \cosh ux + \frac{V(0)}{Z_c} \sinh ux \] (5.131)

We define the following \(ABCD\) parameters:

\[ A(x) = \cosh ux \] (5.132)

\[ B(x) = Z_c \sinh ux \] (5.133)

\[ C(x) = \frac{1}{Z_c} \sinh ux \] (5.134)

\[ D(x) = \cosh ux \] (5.135)

As a result, we have

\[ V(x) = A(x)V(0) + B(x)I(0) \]

\[ I(x) = C(x)V(0) + D(x)I(0) \]

For evaluation of the voltage and current at the sending end \(x = l\), it is common to write

\[ V_s = V(l) \]

\[ I_s = I(l) \]

\[ V_r = V(0) \]

\[ I_r = I(0) \]

Thus we have

\[ V_r = AV_r + BI_r \] (5.136)

\[ I_s = CV_r + DI_r \] (5.137)

The subscripts \(s\) and \(r\) stand for sending and receiving values, respectively. We have from above:

\[ A = A(l) = \cosh ul \] (5.138)

\[ B = B(l) = Z_c \sinh ul \] (5.139)
It is practical to introduce the complex variable \( \theta \) in the definition of the \( ABCD \) parameters. We define

\[
\theta = \frac{\nu l}{\sqrt{Z Y}} \quad (5.142)
\]

As a result,

\[
A = \cosh \theta \quad (5.143)
\]
\[
B = Z_c \sinh \theta \quad (5.144)
\]
\[
C = \frac{1}{Z_c} \sinh \theta \quad (5.145)
\]
\[
D = A \quad (5.146)
\]

Observe that the total line series impedance and admittance are given by

\[
Z = z l \quad (5.147)
\]
\[
Y = y l \quad (5.148)
\]

**Evaluating \( ABCD \) Parameters**

Two methods can be employed to calculate the \( ABCD \) parameters of a transmission line exactly. Both assume that \( \theta \) is calculated in the rectangular form

\[
\theta = \theta_1 + j\theta_2
\]

The first method proceeds by expanding the hyperbolic functions as follows:

\[
A = \frac{e^\theta + e^{-\theta}}{2} = \frac{1}{2} \left( e^{\theta_1} \angle \theta_2 + e^{-\theta_1} \angle -\theta_2 \right)
\]
\[
\sinh \theta = \frac{e^\theta - e^{-\theta}}{2} = \frac{1}{2} \left( e^{\theta_1} \angle \theta_2 - e^{-\theta_1} \angle -\theta_2 \right) \quad (5.149)
\]
\begin{align}
B &= \frac{\sqrt{Z}}{\sqrt{Y}} \sinh \theta \\
C &= \frac{\sqrt{Y}}{\sqrt{Z}} \sinh \theta
\end{align}

(5.150)

(5.151)

Note that \( \theta_2 \) is in radians to start with in the decomposition of \( \theta \).

The second method uses two well-known identities to arrive at the parameter of interest.

\[
A = \cosh(\theta_1 + j\theta_2)
\]

\[
\cosh \theta = \cosh \theta_1 \cos \theta_2 + j \sinh \theta_1 \sin \theta_2
\]

(5.152)

We also have

\[
\sinh \theta = \sinh \theta_1 \cos \theta_2 + j \cosh \theta_1 \sin \theta_2
\]

(5.153)

**Example 5.9**

Find the exact \( ABCD \) parameters for a 235.92-mile long, 735-kV, bundle-conductor line with four subconductors per phase with subconductor resistance of 0.1004 ohms per mile. Assume that the series inductive reactance per phase is 0.5541 ohms per mile and shunt capacitive susceptance of \( 7.4722 \times 10^{-6} \) siemens per mile to neutral. Neglect shunt conductance.

**Solution**

The resistance per phase is

\[
r = \frac{0.1004}{4} = 0.0251 \text{ ohms/mile}
\]

Thus the series impedance in ohms per mile is

\[
z = 0.0251 + j0.5541 \text{ ohms/mile}
\]

The shunt admittance is

\[
y = j7.4722 \times 10^{-6} \text{ siemens/mile}
\]

For the line length,

\[
Z = zl = (0.0251 + j0.5541)(235.92) = 130.86 \angle 87.41^
\]

\[
Y = yl = j(7.4722 \times 10^{-6})(235.92) = 1.7628 \times 10^{-3} \angle 90^
\]

We calculate \( \theta \) as
\[ \theta = \sqrt{Z Y} \]
\[ = \left[ \sqrt{30.86 \angle 87.41^\circ \cdot 1.7628 \times 10^{-3} \angle 90^\circ} \right]^{1/2} \]
\[ = 0.0109 + j0.4802 \]

Thus,

\[ \theta_1 = 0.0109 \]
\[ \theta_2 = 0.4802 \]

We change \( \theta_2 \) to degrees. Therefore,

\[ \theta_2 = (0.4802) \left( \frac{180}{\pi} \right) = 27.5117^\circ \]

Using Eq. (5.149), we then get

\[ \cosh \theta = \frac{1}{2} \left( e^{0.0109} \angle 27.5117^\circ + e^{-0.0109} \angle -27.5117^\circ \right) \]
\[ = 0.8870 \angle 0.3242^\circ \]

From the above,

\[ D = A = 0.8870 \angle 0.3242^\circ \]

We now calculate \( \sinh \theta \) as

\[ \sinh \theta = \frac{1}{2} \left( e^{0.0109} \angle 27.5117^\circ - e^{-0.0109} \angle -27.5117^\circ \right) \]
\[ = 0.4621 \angle 88.8033^\circ \]

We have

\[ Z_c = \sqrt{\frac{Z}{Y}} = \sqrt{\left( \frac{130.86 \angle 87.41^\circ}{1.7628 \times 10^{-3} \angle 90^\circ} \right)} \]
\[ \cdot \left( 74234.17 \angle -2.59 \right)^{1/2} \]
\[ = 272.46 \angle -1.295^\circ \]

As a result,

\[ B = Z_c \sinh \theta \]
\[ = 125.904 \angle 87.508^\circ \]
Also,

\[ C = \frac{1}{Z_c} \sinh \theta \]

\[ = 1.696 \times 10^{-3} \angle 90.098^\circ \]

Let us employ the second method to evaluate the parameters. We find the hyperbolic functions:

\[ \cosh \theta_1 = \cosh(0.0109) \]
\[ = \frac{e^{0.0109} + e^{-0.0109}}{2} \]
\[ = 1.000059 \]

\[ \sinh \theta_1 = \frac{e^{0.0109} - e^{-0.0109}}{2} \]
\[ = 1.09002 \times 10^{-2} \]

(Most calculators have built-in hyperbolic functions, so you can skip the intermediate steps). We also have

\[ \cos \theta_2 = \cos \left(0.4802 \left(\frac{180}{\pi}\right)\right) \]
\[ = 0.8869 \]

\[ \sin \theta_2 = \sin \left(0.4802 \left(\frac{180}{\pi}\right)\right) \]
\[ = 0.4619566 \]

Therefore, we have

\[ \cosh \theta = \cosh \theta_1 \cos \theta_2 + j \sinh \theta_1 \sin \theta_2 \]
\[ = (1.000059)(0.8869) + j(1.09002 \times 10^{-2})(0.4619566) \]
\[ = 0.8869695 \angle 0.32527^\circ \]

\[ \sinh \theta = \sinh \theta_1 \cos \theta_2 + j \cosh \theta_1 \sin \theta_2 \]
\[ = (1.09002 \times 10^{-2})(0.8869) + j(1.000059)(0.4619566) \]
\[ = 0.4620851 \angle 88.801^\circ \]

These results agree with the ones obtained using the first method.

**Example 5.10**

Find the voltage, current, and power at the sending end of the line of Example 5.9 and the transmission efficiency given that the receiving-end load is 1500
MVA at 700 kV with 0.95 PF lagging.

**Solution**

We have the apparent power given by

\[ S_r = 1500 \times 10^6 \text{ VA} \]

The voltage to neutral is

\[ V_r = \frac{700 \times 10^3}{\sqrt{3}} \text{ V} \]

Therefore,

\[ I_r = \frac{1500 \times 10^6}{3 \left( \frac{700 \times 10^3}{\sqrt{3}} \right)} \angle -\cos^{-1} 0.95 \]

\[ = 1237.18 \angle -18.19^\circ \text{ A} \]

From Example 5.9 we have the values of the \(A\), \(B\), and \(C\) parameters. Thus the sending-end voltage (to neutral) is obtained as

\[ V_s = AV_r + BI_r \]

\[ = (0.8870 \angle 0.3253) \left( \frac{700 \times 10^3}{\sqrt{3}} \right) \]

\[ + (25.904 \angle 87.508^\circ)(1237.18 \angle -18.19^\circ) \]

\[ = 439.0938 \angle 19.66^\circ \text{ kV} \]

The line-to-line value is obtained by multiplying the above value by \(\sqrt{3}\), giving

\[ V_{s_L} = 760.533 \text{ kV} \]

The sending-end current is obtained as

\[ I_s = CV_r + DI_r \]

\[ = (1.696 \times 10^{-3} \angle 90.098^\circ) \left( \frac{700 \times 10^3}{\sqrt{3}} \right) \]

\[ + (0.887 \angle 0.3253)(1237.18 \angle -18.19^\circ) \]

\[ = 1100.05 \angle 18.49^\circ \]
The sending-end power factor is

\[ \cos \phi_s = \cos(19.66 - 18.49) = \cos(1.17) = 0.99979 \]

As a result, the sending-end power is

\[ P_s = 3(439.0938 \times 10^3)(1100.05)(0.99979) = 1448.77 \times 10^6 \text{ MW} \]

The efficiency is

\[ \eta = \frac{P_s}{P_l} = \frac{1500 \times 10^6 \times 0.95}{1448.77 \times 10^6} = 0.9836 \]

**Lumped Parameter Transmission Line Models**

Lumped parameter representations of transmission lines are needed for further analysis of interconnected electric power systems. Their use enables the development of simpler algorithms for the solution of complex networks that involve transmission lines.

Here we are interested in obtaining values of the circuit elements of a \( \pi \) circuit, to represent accurately the terminal characteristics of the line given by

\[ V_i = AV_r + BI_r \]
\[ I_i = CV_r + DI_r \]

It is easy to verify that the elements of the equivalent circuit are given in terms of the \( ABCD \) parameters of the line by

\[ Z_\pi = B \quad (5.154) \]

and

\[ Y_\pi = \frac{A-1}{B} \quad (5.155) \]

The circuit is shown in Figure 5.25.
Example 5.11

Find the equivalent π-circuit elements for the line of Example 5.9.

Solution

From Example 5.9, we have

\[ A = 0.8870 \angle 0.3242^\circ \]
\[ B = 125.904 \angle 87.508^\circ \]

As a result, we have

\[ Z_\pi = 125.904 \angle 87.508^\circ \text{ ohms} \]
\[ Y_\pi = \frac{0.8870 \angle 0.3242^\circ - 1}{125.904 \angle 87.508^\circ} \]
\[ = 8.9851 \times 10^{-4} \angle 89.941^\circ \text{ siemens} \]

The following MATLAB™ Script implements Examples 5.9, 5.10, and 5.11

```matlab
% Example 5-9
% To enter the data
r=0.0251;
x=0.5541;
l=235.92;
y=i*7.4722*10^-6;
Sr=1500*10^6;
Vr=(700*10^3)/3^.5;
% for the line length
z=r+i*x;
Z=z*l;
Y=y*l;
% To calculate theta
theta=(Z*Y)^.5;
theta2=imag(theta)*180/pi;
```
MATLAB™ con’t.

```matlab
% To calculate A and D
D = cosh(theta)
A = D
A_mod = abs(A)
delta = angle(A) * 180/pi
% To calculate B and C
Zc = (Z/Y)^(1/2);
B = Zc * sinh(theta)
B_mod = abs(B)
delta1 = angle(B) * 180/pi
C = 1/Zc * sinh(theta)
C_mod = abs(C)
delta2 = angle(C) * 180/pi
% To evaluate the parameters.
% We find the hyperbolic functions
cosh(real(theta));
sinh(real(theta));
%
% Example 5-10
%
Ir = Sr / (3*Vr);
% power factor 0.95 lagging
alpha = acos(0.95);
alpha_deg = alpha * 180/pi;
Pr = Sr * cos(alpha);
Ir_compl = Ir * (cos(-alpha) + i*sin(-alpha));
% To calculate sending end voltage (to neutral)
Vs = A*Vr + B*Ir_compl
Vs_mod = abs(Vs)
Vs_arg = angle(Vs) * 180/pi
% line to line voltage
Vsl = Vs^(3/2).
% To calculate sending end current
Is = C*Vr + D*Ir_compl
Is_mod = abs(Is)
Is_arg = angle(Is) * 180/pi
% To calculate sending end power factor
pf_sending = cos(angle(Vs) - angle(Is))
% To calculate sending end power
Ps = 3*abs(Vs)*abs(Is)*pf_sending
% To calculate the efficiency
eff = Pr/Ps
%```

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MATLAB™ con’t.

```matlab
% example 5-11
% To find the equivalent pi-circuit elements
Zpi = B
Zpi_mod = abs(Zpi)
Zpi_arg = angle(Zpi) * 180 / pi
Ypi = (A - 1) / B
Ypi_mod = abs(Ypi)
Ypi_arg = angle(Ypi) * 180 / pi
```

The results of running the script are as shown below:

```
EDU»
D = 0.8870 + 0.0050i
A = 0.8870 + 0.0050i
A_mod = 0.8870
delta = 0.3244
B = 5.4745e+000 + 1.2577e+002i
B_mod = 125.8891
delta1 = 87.5076
C = 0.0000 + 0.0017i
C_mod = 0.0017
delta2 = 90.1012
Vs = 4.1348e+005 + 1.4773e+005i
```

**Approximations to the ABCD Parameters of Transmission Lines**

Consider the series expansion of the hyperbolic functions defining the $A$, $B$, $C$, and $D$ parameters using $\theta = \sqrt{Z\gamma}$.

Usually no more than three terms are required. For overhead lines less than 500 km in length, the following approximate expressions are satisfactory:

$$A = D = 1 + \frac{Z\gamma}{2} \quad (5.156)$$

$$B = Z \left(1 + \frac{Z\gamma}{6}\right) \quad (5.157)$$
Figure 5.26 Nominal π Model of a Medium Transmission Line.

\[ C = Y \left( 1 + \frac{ZY}{6} \right) \]  

(5.158)

If only the first term of the expansions is used, then

\[ B = Z \]  

(5.159)

\[ \frac{A-1}{B} = \frac{Y}{2} \]  

(5.160)

In this case, the equivalent π circuit reduces to the nominal π, which is used generally for lines classified as medium lines (up to 250 km). Figure 5.26 shows the nominal π model of a medium transmission line. The result we obtained analytically could have been obtained easily by the intuitive assumption that the line’s series impedance is lumped together and the shunt admittance \( Y \) is divided equally with each half placed at each end of the line.

A final model is the short-line (up to 80 km) model, and in this case the shunt admittance is neglected altogether. The line is thus represented only by its series impedance.

**Example 5.12**

Find the nominal π and short-line representations for the line of Example 5.9. Calculate the sending-end voltage and current of the transmission line using the two representations under the conditions of Example 5.10.

**Solution**

For this line we have

\[ Z = 130.86 \angle 87.41^\circ \]

\[ Y = 1.7628 \times 10^{-3} \angle 90^\circ \]
As a result, we have the representations shown in Figure 5.26.

From Example 5.10, we have

\[ V_r = \frac{700 \times 10^3}{\sqrt{3}} \text{ V} \]

\[ I_r = 1237.18 \angle -18.19^\circ \text{ A} \]

For the short-line representation we have

\[ V_s = V_r + I_r Z \]
\[ = \frac{700 \times 10^3}{\sqrt{3}} + (1237.18 \angle -18.19^\circ)(130.86 \angle 87.41^\circ) \]
\[ = 485.7682 \times 10^3 \angle 18.16^\circ \text{ V} \]

For the nominal \( \pi \) we have

\[ I_L = I_r + V_r \left( \frac{Y}{2} \right) \]
\[ = (1237.18 \angle -18.19^\circ) + \frac{700 \times 10^3}{\sqrt{3}} \left(0.8814 \times 10^{-3} \angle 90^\circ\right) \]
\[ = 1175.74 \angle -1.4619^\circ \text{ A} \]

Thus,

\[ V_s = V_r + I_r Z \]
\[ = \frac{700 \times 10^3}{\sqrt{3}} + (1175.74 \angle -1.4619^\circ)(130.86 \angle 87.41^\circ) \]
\[ = 442.484 \times 10^3 \angle 20.2943^\circ \text{ V} \]

Referring back to the exact values calculated in Example 5.10, we find that the short-line approximation results in an error in the voltage magnitude of

\[ \Delta V = \frac{439.0938 - 485.7682}{439.0938} \]
\[ = -0.11 \]

For the nominal \( \pi \) we have the error of
\[ \Delta V = \frac{439.0938 - 442.484}{439.0938} = -0.00772 \]

which is less than 1 percent.

The sending-end current with the nominal π model is

\[ I_s = I_L + V_s \left( \frac{Y}{2} \right) \]

\[ = 1175.74 \angle -1.4619^\circ \]

\[ + (442.484 \angle 20.2943^\circ)(0.8814 \times 10^3 \angle 90^\circ) \]

\[ = 1092.95 \angle 17.89^\circ \text{ A} \]

The following MATLAB™ Script implements Example 5.12

```matlab
% Example 5-12
% From example 5-9, we have
Z = 130.86 * (cos(87.41 * pi/180) + i * sin(87.41 * pi/180));
Y = i * 1.7628 * 10^-3;
% From example 5-10, we have
Vr = 700 * 10^3 / (3^.5);
Ir = 1237.18 * (cos(-18.19 * pi/180) + i * sin(-18.19 * pi/180));
% For the short-line representation we have
Vs = Vr + Ir * Z;
Vs_mod = abs(Vs)
Vs_arg = angle(Vs) * 180 / pi
% for the nominal pi, we have
IL = Ir + Vr * (Y / 2);
IL_mod = abs(IL)
IL_arg = angle(IL) * 180 / pi
% Thus
Vs = Vr + IL * Z;
Vs_mod = abs(Vs)
Vs_arg = angle(Vs) * 180 / pi
% The sending-end current with the nominal pi model is
Is = IL + Vs * (Y / 2)
Is_mod = abs(Is)
Is_arg = angle(Is) * 180 / pi
```
**PROBLEMS**

**Problem 5.1**
Determine the inductive reactance in ohms/mile/phase for a 345-kV, single-circuit line with ACSR 84/19 conductor for which the geometric mean radius is 0.0588 ft. Assume a horizontal phase configuration with 26-ft phase separation.

**Problem 5.2**
Calculate the inductive reactance in ohms/mile/phase for a 500-kV, single-circuit, two-subconductor bundle line with ACSR 84/19 subconductor for which the GMR is 0.0534 ft. Assume horizontal phase configuration with 33.5-ft phase separation. Assume bundle separation is 18 in.

**Problem 5.3**
Repeat Problem 5.2 for a phase separation of 35 ft.

**Problem 5.4**
Repeat Problem 5.3 with an ACSR 76/19 subconductor for which the GMR is 0.0595 ft.

**Problem 5.5**
Find the inductive reactance in ohms/mile/phase for a 500-kV, single-circuit, two-subconductor bundle line with ACSR 84/19 conductor for which the GMR is 0.0534 ft. Assume horizontal phase configuration with separation of 32 ft. Bundle spacing is 18 in.

**Problem 5.6**
Find the inductive reactance in ohms/mile/phase for the 765-kV, single-circuit, bundle-conductor line with four subconductors per bundle at a spacing of 18 in., given that the subconductor GMR is 0.0385 ft. Assume horizontal phase configuration with 44.5-ft phase separation.
Problem 5.7
Repeat Problem 5.6 for bundle spacing of 24 in. and subconductor GMR of 0.0515 ft. Assume phase separation is 45 ft.

Problem 5.8
Calculate the inductance in henries per meter per phase for the 1100-kV, bundle-conductor line shown in Figure 5.27. Assume phase spacing $D_1 = 15.24$ m, bundle separation $S = 45.72$ cm, and conductor diameter is 3.556 cm.

Figure 5.27 Line for Problem 5.8.

Problem 5.9
Calculate the inductive reactance in ohms per mile for the 500-kV, double-circuit, bundle-conductor line with three subconductors of 0.0431-ft GMR and with 18-in. bundle separation. Assume conductor configurations as shown in Figure 5.28.

Problem 5.10
Calculate the inductive reactance in ohms per mile for 345-kV, double-circuit, bundle-conductor line with two subconductors per bundle at 18-in. bundle spacing. Assume subconductor’s GMR is 0.0373 ft, and conductor configuration is as shown in Figure 5.29.

Problem 5.11
Calculate the inductive reactance in ohms per mile for the 345-kV double-circuit, bundle-conductor line with two subconductors per bundle at 18-in. bundle spacing. Assume subconductor’s GMR is 0.0497 ft, and conductor configuration is as shown in Figure 5.30.
**Problem 5.12**
Determine the capacitive reactance in ohm miles for the line of Problem 5.1. Assume the conductor’s outside diameter is 1.76 in. Repeat by including earth effects given that the ground clearance is 45 ft.

**Problem 5.13**
Determine the capacitive reactance in ohm miles for the line of Problem 5.2. Assume the conductor’s outside diameter is 1.602 in. Repeat by including earth effects given that the ground clearance is 82 ft.
Problem 5.14
Determine the capacitive reactance in ohm miles for the line of Problem 5.3. Assume the conductor’s outside diameter is 1.602 in. Repeat by including earth effects given that the ground clearance is 136 ft.

Problem 5.15
Determine the capacitive reactance in ohm miles for the line of Problem 5.4. Assume the conductor’s outside diameter is 1.7 in. Neglect earth effects.

Problem 5.16
Determine the capacitive reactance in ohm miles for the line of Problem 5.5. Assume the conductor’s outside diameter is 1.762 in. Repeat by including earth effects given that the ground clearance is 63 ft.

Problem 5.17
Determine the capacitive reactance in ohm miles for the line of Problem 5.6. Assume the conductor’s outside diameter is 1.165 in.

Problem 5.18
Determine the capacitive reactance in ohm miles for the line of Problem 5.7. Assume the conductor’s outside diameter is 1.6 in. Repeat by including earth effects given that the ground clearance is 90 ft.

Problem 5.19
Calculate the capacitance in farads per meter per phase neglecting earth effect for the 1100-kV, bundle-conductor line of Problem 5.8. Assume the conductor diameter is 3.556 cm. Repeat including earth effects with \( h_1 = 21.34 \) m.

Problem 5.20
Determine the capacitive reactance in ohm mile for the line of Problem 5.9. Assume the conductor’s outside diameter is 1.302 in. Neglect earth effect.

Problem 5.21
Determine the capacitive reactance in ohm mile for the line of Problem 5.10. Assume the conductor’s outside diameter is 1.165 in.

Problem 5.22
Determine the capacitive reactance in ohm mile for the line of Problem 5.11. Assume the conductor’s outside diameter is 1.302 in.

Problem 5.23
Assume that the 345-kV line of Problems 5.1 and 5.12 is 14 miles long and that the conductor’s resistance is 0.0466 ohms/mile.

A. Calculate the exact \( ABCD \) parameters for the line.
B. Find the circuit elements of the equivalent \( \pi \) model for the line. Neglect earth effects.
Problem 5.24
Assume that the 1100-kV line of Problems 5.8 and 5.19 is 400 km long and that
the subconductor’s resistance is 0.0435 ohms/km.

A. Calculate the exact ABCD parameters for the line.
B. Find the circuit elements of the equivalent \( \pi \) model for the line. 
   Neglect earth effects.

Problem 5.25
The following information is available for a single-circuit, three-phase, 345-kV,
360 mega volt amperes (MVA) transmission line:

- Line length = 413 miles.
- Number of conductors per phase = 2.
- Bundle spacing = 18 in.
- Outside conductor diameter = 1.165 in.
- Conductor’s GMR = 0.0374 ft.
- Conductor’s resistance = 0.1062 ohms/mile.
- Phase separation = 30 ft.
- Phase configuration is equilateral triangle.
- Minimum ground clearance = 80 ft.

A. Calculate the line’s inductive reactance in ohms per mile per phase.
B. Calculate the capacitive reactance including earth effects in ohm
   miles per phase.
C. Calculate the exact A and B parameters of the line.
D. Find the voltage at the sending end of the line if normal rating
   power at 0.9 PF is delivered at 345-kV at the receiving end. Use
   the exact formulation.
E. Repeat (d) using the short-line approximation. Find the error
   involved in computing the magnitude of the sending-end voltage
   between this method and the exact one.

Problem 5.26
For the transmission line of Problem 5.24, calculate the sending-end voltage,
sending-end current, power, and power factor when the line is delivering 4500
MVA at 0.9 PF lagging at rated voltage, using the following:

A. Exact formulation.
B. Nominal \( \pi \) approximation.
C. Short-line approximation.