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Chapter 3

POWER GENERATION AND
THE SYNCHRONOUS MACHINE

3.1 INTRODUCTION

The backbone of any electric power system is a number of generating stations operating in parallel. At each station there may be several synchronous generators operating in parallel. Synchronous machines represent the largest single-unit electric machine in production. Generators with power ratings of several hundred to over a thousand megavoltamperes (MVA) are fairly common in many utility systems. A synchronous machine provides a reliable and efficient means for energy conversion.

The operation of a synchronous generator is (like all other electromechanical energy conversion devices) based on Faraday’s law of electromagnetic induction. The term synchronous refers to the fact that this type of machine operates at constant speed and frequency under steady-state conditions. Synchronous machines are equally capable of operating as motors, in which case the electric energy supplied at the armature terminals of the unit is converted into mechanical form.

3.2 THE SYNCHRONOUS MACHINE: PRELIMINARIES

The armature winding of a synchronous machine is on the stator, and the field winding is on the rotor as shown in Figure 3.1. The field is excited by the direct current that is conducted through carbon brushes bearing on slip (or collector) rings. The dc source is called the exciter and is often mounted on the same shaft as the synchronous machine. Various excitation systems with ac exciters and solid-state rectifiers are used with large turbine generators. The main advantages of these systems include the elimination of cooling and maintenance problems associated with slip rings, commutators, and brushes. The pole faces are shaped such that the radial distribution of the air-gap flux density B is approximately sinusoidal as shown in Figure 3.2.

The armature winding will include many coils. One coil is shown in Figure 3.1 and has two coil sides (a and –a) placed in diametrically opposite slots on the inner periphery of the stator with conductors parallel to the shaft of the machine. The rotor is turned at a constant speed by a power mover connected to its shaft. As a result, the flux waveform sweeps by the coil sides a and –a. The induced voltage in the coil is a sinusoidal time function. For each revolution of the two poles, the coil voltage passes through a complete cycle of values. The frequency of the voltage in cycles per second (hertz) is the same as the rotor speed in revolutions per second. Thus, a two-pole synchronous machine must revolve at 3600 r/min to produce a 60-Hz voltage.
Many synchronous machines have more than two poles. A P-pole machine is one with P poles. As an example, we consider an elementary, single-phase, four-pole generator shown in Figure 3.3. There are two complete cycles in the flux distribution around the periphery as shown in Figure 3.4. The armature winding in this case consists of two coils \((a_1, -a_1, a_2, -a_2)\) connected in series. The generated voltage goes through two complete cycles per revolution of the rotor, and thus the frequency \(f\) in hertz is twice the speed in revolutions per second. In general, the coil voltage of a machine with \(P\)-poles passes through a complete cycle every time a pair of poles sweeps by, or \(P/2\) times for each revolution. The frequency \(f\) is therefore given by

\[
 f = \frac{P}{2} \left( \frac{n}{60} \right)
\]

where \(n\) is the shaft speed in revolutions per minute (r/min).

In treating \(P\)-pole synchronous machines, it is more convenient to express angles in electrical degrees rather than in the more familiar mechanical units. Here we concentrate on a single pair of poles and recognize that the conditions associated with any other pair are simply repetitions of those of the pair under consideration. A full cycle of generated voltage will be described when the rotor of a four-pole machine has turned 180 mechanical degrees. This

**P-Pole Machines**
cycle represents 360 electrical degrees in the voltage wave. Extending this argument to a $P$-pole machine leads to

$$\theta_e = \frac{P}{2} \theta_m$$

where $\theta_e$ and $\theta_m$ denote angles in electrical and mechanical degrees, respectively.

Cylindrical vs. Salient-Pole Construction

Machines like the ones illustrated in Figures 3.1 and 3.3 have rotors with salient poles. There is another type of rotor, which is shown in Figure 3.5. The machine with such a rotor is called a cylindrical rotor or nonsalient-pole machine. The choice between the two designs (salient or nonsalient) for a specific application depends on the prime mover. For hydroelectric generation, a salient-pole construction is employed, because hydraulic turbines run at relatively low speeds, and a large number of poles is required to produce the desired frequency as indicated by Eq. (3.1). Steam and gas turbines perform better at relatively high speeds, and two- or four-pole cylindrical rotor turboalternators are used to avoid the use of protruding parts on the rotor.
3.3 SYNCHRONOUS MACHINE FIELDS

An understanding of the nature of the magnetic field produced by a polyphase winding is necessary for the analysis of polyphase ac machines. We will consider a two-pole, three-phase machine. The windings of the individual phases are displaced by 120 electrical degrees in space. The magnetomotive forces developed in the air gap due to currents in the windings will also be displaced 120 electrical degrees in space. Assuming sinusoidal, balanced three-phase operation, the phase currents are displaced by 120 electrical degrees in time.

Assume that \( I_m \) is the maximum value of the current, and the time origin is arbitrarily taken as the instant when the phase \( a \) current is a positive maximum. The phase sequence is assumed to be \( abc \).

The magnetomotive force (MMF) of each phase is proportional to the corresponding current, and hence, the peak MMF is given by

\[
F_{\text{max}} = K I_m
\]

where \( K \) is a constant of proportionality that depends on the winding distribution and the number of series turns in the winding per phase. We thus have

\[
A_a(p) = F_{\text{max}} \cos(\omega t)
\]

(3.2)

\[
A_b(p) = F_{\text{max}} \cos(\omega t - 120^\circ)
\]

(3.3)

\[
A_c(p) = F_{\text{max}} \cos(\omega t - 240^\circ)
\]

(3.4)

where \( A_a(p) \) is the amplitude of the MMF component wave at time \( t \).
At time \( t \), all three phases contribute to the air-gap MMF at a point \( P \) (whose spatial angle is \( \theta \)). The resultant MMF is then given by

\[
A_p = A_{a(p)} \cos \theta + A_{b(p)} \cos(\theta - 120^\circ) + A_{c(p)} \cos(\theta - 240^\circ) \tag{3.5}
\]

This reduces to

\[
A_p = \frac{3}{\tau} [F_{\text{max}} \cos(\theta - \omega t)] \tag{3.6}
\]

The wave represented in Eq. (3.6) depends on the spatial position \( \theta \) as well as time. The angle \( \omega t \) provides rotation of the entire wave around the air gap at the constant angular velocity \( \omega \). At time \( t_1 \), the wave is a sinusoid with its positive peak displaced \( \omega t_1 \) from the point \( P \) (at \( \theta \)); at a later instant (\( t_2 \)) the wave has its positive peak displaced \( \omega t_2 \) from the same point. We thus see that a polyphase winding excited by balanced polyphase currents produces the same effect as a permanent magnet rotating within the stator.

The MMF wave created by the three-phase armature current in a synchronous machine is commonly called armature-reaction MMF. It is a wave that rotates at synchronous speed and is directly opposite to phase \( a \) at the instant when phase \( a \) has its maximum current \( (t = 0) \). The dc field winding produces a sinusoid \( F \) with an axis 90° ahead of the axis of phase \( a \) in accordance with Faraday’s law.

The resultant magnetic field in the machine is the sum of the two contributions from the field and armature reaction. Figure 3.6 shows a sketch of the armature and field windings of a cylindrical rotor generator. The space MMF produced by the field winding is shown by the sinusoid \( F \). This is shown for the specific instant when the electromotive force (EMF) of phase \( a \) due to excitation has its maximum value. The time rate of change of flux linkages with phase \( a \) is a maximum under these conditions, and thus the axis of the field is 90° ahead of phase \( a \). The armature-reaction wave is shown as the sinusoid \( A \) in the figure. This is drawn opposite phase \( a \) because at this instant both \( I_a \) and the EMF of the field \( E_f \) (also called excitation voltage) have their maximum value. The resultant magnetic field in the machine is denoted \( R \) and is obtained by graphically adding the \( F \) and \( A \) waves.

Sinusoids can conveniently be handled using phasor methods. We can thus perform the addition of the \( A \) and \( F \) waves using phasor notation. Figure 3.7 shows a space phasor diagram where the fluxes \( \phi_f \) (due to the field), \( \phi_{ar} \) (due to armature reaction), and \( \phi_r \) (the resultant flux) are represented. It is clear that under the assumption of a uniform air gap and no saturation, these are proportional to the MMF waves \( F, A, \) and \( R \), respectively. The figure is drawn for the case when the armature current is in phase with the excitation voltage.
3.4 A SIMPLE EQUIVALENT CIRCUIT

The simplest model of a synchronous machine with cylindrical rotor can be obtained if the effect of the armature-reaction flux is represented by an inductive reactance. The basis for this is shown in Figure 3.8, where the phasor diagram of component fluxes and corresponding voltages is given. The field flux $\phi_f$ is added to the armature-reaction flux $\phi_{ar}$ to yield the resultant air-gap flux $\phi_r$. The armature-reaction flux $\phi_{ar}$ is in phase with the armature current $I_a$. The excitation voltage $E_f$ is generated by the field flux, and $E_f$ lags $\phi_f$ by 90°. Similarly, $E_{ar}$ and $E_r$ are generated by $\phi_{ar}$ and $\phi_r$, respectively, with each of the voltages lagging the flux causing it by 90°.

Introduce the constant of proportionality $x_\phi$ to relate the rms values of $E_{ar}$ and $I_a$, to write
Figure 3.8 Phasor Diagram for Fluxes and Resulting Voltages in a Synchronous Machine.

Figure 3.9 Two Equivalent Circuits for the Synchronous Machine.

\[ E_{ar} = -jx_\phi I_a \]  

(3.7)

where the \(-j\) represents the 90° lagging effect. We therefore have

\[ E_r = E_f - jx_\phi I_a \]  

(3.8)

An equivalent circuit based on Eq. (3.8) is given in Figure 3.9. We thus conclude that the inductive reactance \(x_\phi\) accounts for the armature-reaction effects. This reactance is known as the magnetizing reactance of the machine.

The terminal voltage of the machine denoted by \(V_t\) is the difference between the air-gap voltage \(E_r\) and the voltage drops in the armature resistance \(r_a\), and the leakage-reactance \(x_l\). Here \(x_l\) accounts for the effects of leakage flux as well as space harmonic filed effects not accounted for by \(x_\phi\). A simple impedance commonly known as the synchronous impedance \(Z_s\) is obtained by combining \(x_\phi\), \(x_l\), and \(r_a\) according to

\[ Z_s = r_a + jX_s \]  

(3.9)

The synchronous reactance \(X_s\) is given by

\[ X_s = x_l + x_\phi \]  

(3.10)

The model obtained here applies to an unsaturated cylindrical rotor machine supplying balanced polyphase currents to its load. The voltage relationship is now given by
Example 3.1
A 10 MVA, 13.8 kV, 60 Hz, two-pole, Y-connected, three-phase alternator has an armature winding resistance of 0.07 ohms per phase and a leakage reactance of 1.9 ohms per phase. The armature reaction EMF for the machine is related to the armature current by

\[ E_{ar} = -j19.91I_a \]

Assume that the generated EMF is related to the field current by

\[ E_f = 60I_f \]

A. Compute the field current required to establish rated voltage across the terminals of a load when rated armature current is delivered at 0.8 PF lagging.

B. Compute the field current needed to provide rated terminal voltage to a load that draws 100 per cent of rated current at 0.85 PF lagging.

Solution
The rated current is given by

\[ I_a = \frac{10 \times 10^6}{\sqrt{3} \times 13800} = 418.37 \text{ A} \]

The phase value of terminal voltage is

\[ V_t = \frac{13.800}{\sqrt{3}} = 7967.43 \text{ V} \]

With reference to the equivalent circuit of Figure 3.9, we have

A.

\[ E_r = V_t + I_a Z_a = 7967.43 + (418.37 \angle \cos^{-1} 0.8)(0.07 + j1.9) \]
\[ = 8490.35 \angle 4.18^\circ \]
\[ E_{ar} = -j(19.91)(418.37 \angle \cos^{-1} 0.8) = -8329.75 \angle 53.13^\circ \]

The required field excitation voltage \( E_f \) is therefore,
\[ E_f = E_r - E_{ar} \]
\[ = 8490.35 \angle 4.18\,^\circ + 8329.75 \angle 53.13\,^\circ \]
\[ = 15308.61 \angle 28.4^\circ \text{ V} \]

Consequently, using the given field voltage versus current relation,

\[ I_f = \frac{E_f}{60} = 255.14 \text{ A} \]

**B. With conditions given, we have**

\[ I_a = (418.37)(\cos^{-1} 0.85) = 418.37 \angle -31.79^\circ \]
\[ E_r = 7967.43 + (418.37 \angle -31.79^\circ)(0.07 + j1.9) \]
\[ = 8436.94 \angle 4.49^\circ \text{ V} \]
\[ E_{ar} = -j(19.91)(418.37 \angle -31.79^\circ) \]
\[ = -8329.74 \angle 58.21^\circ \]
\[ E_f = E_r - E_{ar} \]
\[ = 8436.94 \angle 4.48^\circ + 8329.74 \angle 58.21^\circ \]
\[ = 14,957.72 \angle 31.16^\circ \text{ V} \]

We therefore calculate the required field current as

\[ I_f = \frac{14,957.72}{60} = 249.30 \text{ A} \]

### 3.5 PRINCIPAL STEADY-STATE CHARACTERISTICS

Consider a synchronous generator delivering power to a constant power factor load at a constant frequency. A *compounding curve* shows the variation of the field current required to maintain rated terminal voltage with the load. Typical compounding curves for various power factors are shown in Figure 3.10. The computation of points on the curve follows easily from applying Eq. (3.11). Figure 3.11 shows phasor diagram representations for three different power factors.

**Example 3.2**

A 1,250-kVA, three-phase, Y-connected, 4,160-V (line-to-line), ten-pole, 60-Hz generator has an armature resistance of 0.126 ohms per phase and a synchronous reactance of 3 ohms per phase. Find the full load generated voltage per phase at a power factor of 0.8 lagging.
Solution

The magnitude of full load current is obtained as

$$I_a = \frac{1.250 \times 10^3}{\sqrt{3} \times 4.160} = 173.48 \text{ A}$$

The terminal voltage per phase is taken as reference

$$V_t = \frac{4.160}{\sqrt{3}} = 2401.77 \angle 0 \text{ V}$$

The synchronous impedance is obtained as

$$Z_s = r_a + jX_s$$
$$= 0.126 + j3$$
$$= 3.0026 \angle 87.59^\circ \text{ ohms per phase}$$

The generated voltage per phase is obtained using Eq. (3.11) as:

For a power factor of 0.8 lagging: $\phi = -36.87^\circ$.

$$I_a = 173.48 \angle -36.87^\circ \text{ A}$$
$$E_f = 2.40177 + \left(173.48 \angle -36.87^\circ \times 3.0026 \angle 87.59^\circ\right)$$
$$= 2761.137 \angle 8.397^\circ \text{ V}$$

A characteristic of the synchronous machine is given by the reactive-capability curves. These give the maximum reactive power loadings corresponding to various active power loadings for rated voltage operation. Armature heating constraints govern the machine for power factors from rated to unity. Field heating represents the constraints for lower power factors. Figure 3.12 shows a typical set of curves for a large turbine generator.

Figure 3.10 Synchronous-Machine Compounding Curves.
Figure 3.11 Phasor Diagrams for a Synchronous Machine Operating at Different Power Factors are:
(a) Unity PF Loads, (b) Lagging PF Loads, and (c) Leading PF Loads.

3.6 POWER-ANGLE CHARACTERISTICS AND THE INFINITE BUS CONCEPT

Consider the simple circuit shown in Figure 3.13. The impedance $Z$ connects the sending end, whose voltage is $E$ and receiving end, with voltage $V$. Let us assume that in polar form we have

\[
E = E \angle \delta \\
V = V \angle 0 \\
Z = Z \angle \psi
\]

We therefore conclude that the current $I$ is given by

\[
I = \frac{E - V}{Z}
\]

The complex power $S_1$ at the sending end is given by

\[
S_1^* = E^* I
\]

Similarly, the complex power $S_2$ at the receiving end is
\[ S_1^* = V^*I \]

Therefore,

\[ S_1^* = \frac{E^2}{Z} - \psi - \frac{EV}{Z} - \psi - \delta \]  \hspace{1cm} (3.12)  

\[ S_2^* = \frac{EV}{Z} - \delta - \psi - \frac{V^2}{Z} - \psi \]  \hspace{1cm} (3.13)  

Recall that

\[ S^* = P - jQ \]

When the resistance is negligible; then
\[ \psi = 90^\circ \]
\[ Z = X \]

and the power equations are obtained as:

\[ P_1 = P_2 = \frac{E V}{X} \sin \delta \quad (3.14) \]

\[ Q_1 = \frac{E^2 - EV \cos \delta}{X} \quad (3.15) \]

\[ Q_2 = \frac{EV \cos \delta - V^2}{X} \quad (3.16) \]

In large-scale power systems, a three-phase synchronous machine is connected through an equivalent system reactance \( X_e \) to the network which has a high generation capacity relative to any single unit. We often refer to the network or system as an infinite bus when a change in input mechanical power or in field excitation to the unit does not cause an appreciable change in system frequency or terminal voltage. Figure 3.14 shows such a situation, where \( V \) is the infinite bus voltage.

The previous analysis shows that in the present case we have for power transfer,

\[ P = P_{\text{max}} \sin \delta \quad (3.17) \]

with

\[ P_{\text{max}} = \frac{E V}{X_e} \quad (3.18) \]

and

\[ X_e = X_s + X_e \quad (3.19) \]

If we try to advance \( \delta \) further than \( 90^\circ \) (corresponding to maximum power transfer) by increasing the mechanical power input, the electrical power output would decrease from the \( P_{\text{max}} \) point. Therefore the angle \( \delta \) increases further as the machine accelerates. This drives the machine and system apart electrically. The value \( P_{\text{max}} \) is called the steady-state stability limit or pull-out power.

**Example 3.3**
A synchronous generator with a synchronous reactance of 1.15 p.u. is connected
to an infinite bus whose voltage is one p.u. through an equivalent reactance of 0.15 p.u. The maximum permissible output is 1.2 p.u.

A. Compute the excitation voltage $E$.
B. The power output is gradually reduced to 0.7 p.u. with fixed field excitation. Find the new current and power angle $\delta$.

**Solution**

A. The total reactance is

$$X_t = 1.15 + 0.15 = 1.3$$

Thus we have,

$$1.2 = \frac{EV}{X_t}$$

$$= \frac{(E)(1)}{1.3}$$

Therefore,

$$E = 1.56 \text{ p.u.}$$

B. We have for any angle $\delta$,

$$P = P_{\text{max}} \sin \delta$$

Therefore,

$$0.7 = 1.2 \sin \delta$$

This results in

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The current is

\[ I = \frac{E - V}{jX_t} \]

Substituting the given values, we obtain

\[ I = \frac{1.56 \angle 35.69^\circ - 1.0}{j1.3} = 0.7296 \angle -16.35^\circ \text{ A} \]

The following is a MATLAB™ script to solve problems of the type presented in Example 3.3.

```matlab
% example 3.3
% enter the data
Xs=1.25; % synchronous reactance
Xe=0.25; % equivalent reactance
Pm=1.2; % max permissible output
V=1; % infinite bus voltage
% to find the total reactance
Xt=Xs+Xe;
% A. To compute the exitation voltage
% from Pm=E*V/Xt
E=Pm*Xt/V
% B. The power output is gradually reduced to 0.7 p.u.
% with fixed field excitation.
% to find power angle delta
P=0.7; % power output
% from P=Pm*sin(delta)
delta=asin(P/Pm);
delta_deg=delta*180/pi
E_complex=E*(cos(delta)+i*sin(delta));
% To find the new current
% modulus and argumen
I=(E_complex-V)/Xt*i;
modulus_I=abs(I)
eta=atan(imag(I)/real(I));
argumen_I=eta*180/pi
```
The solution is obtained by running the script as follows

```plaintext
EDU»
E = 1.5600
delta_deg = 35.6853
modulus_I = 0.7296
argumen_I = -16.3500°
```

**Reactive Power Generation**

Eq. (3.16) suggests that the generator produces reactive power \( Q_2 > 0 \) if

\[
E \cos \delta > V
\]

In this case, the generator appears to the network as a capacitor. This condition applies for high magnitude \( E \), and the machine is said to be overexcited. On the other hand, the machine is underexcited if it consumes reactive power \( Q_2 < 0 \). Here we have

\[
E \cos \delta < V
\]

Figure 3.15 shows phasor diagrams for both cases. The overexcited synchronous machine is normally employed to provide synchronous condenser action, where usually no real load is carried by the machine \( (\delta = 0) \). In this case we have

\[
Q_2 = \frac{V(E - V)}{X} \tag{3.20}
\]

Control of reactive power generation is carried out by simply changing \( E \), by varying the dc excitation.

**Example 3.4**

Compute the reactive power generated by the machine of Example 3.3 under the conditions in part (b). If the machine is required to generate a reactive power of 0.4 p.u. while supplying the same active power by changing the filed excitation, find the new excitation voltage and power angle \( \delta \).

**Solution**

The reactive power generated is obtained according to Eq. (3.16) as

\[ Q_2 = \frac{1(1.56 \cos 35.69 - 1)}{1.3} = 0.205 \]

With a new excitation voltage and stated active and reactive powers, we have
using Eq. (3.14) and (3.16)

\[
0.7 = \frac{(E)(1)}{(1.3)} \sin \delta \\
0.4 = \frac{1(E \cos \delta - 1)}{1.3}
\]

We thus obtain

\[
\tan \delta = \frac{(1.3)(0.7)}{(1.52)}
\]

\[
\delta = 30.9083°
\]

From the above we get

\[
E = \frac{(1.3)(0.7)}{\sin(30.9083°)} = 1.7716
\]

Figure 3.15 Phasor Diagrams for Overexcited and Underexcited Synchronous Machines.
The following script implements the solution of this example in MATLAB™ environment.

```matlab
% example 3.4
% enter the data
Xs=1.15; % synchronous reactance
Xe=0.15; % equivalent reactance
Pm=1.2; % max permissible output
V=1; % infinite bus voltage
%
% to find the total reactance
Xt=Xs+Xe;
% A. To compute the exitation voltage
% from Pm=E*V/Xt
E=Pm*Xt/V;
P=0.7; % power output
% from P=Pm*sin(delta)
delta=asin(P/Pm);
%
% to compute reactive power generated
Q2=(E*V*cos(delta)-V^2)/Xt;
% If the machine is required to
% of 0.4 p.u. while supplying the same
active power
% to find the new power angle (delta1)
Q2_required=0.4;
% with a new excitation voltage
% and stated active and reactive powers
% using the equation
% P=(E*V/Xt)sin(delta1) and
Q2=(E*V*cos(delta1)-V^2)/Xt
delta1=atan(P/(Q2_required+V^2/Xt));
delta1_deg=delta1*180/pi
% to find the new field exitation
E_new=P*Xt/sin(delta1)
```

The solution is obtained as

```matlab
EDU»
delta1_deg = 30.9083
E_new = 1.7716
```
3.7 ACCOUNTING FOR SALIENCY

Field poles in a salient-pole machine cause nonuniformity of the magnetic reluctance of the air gap. The reluctance along the polar axis is appreciably less than that along the interpolar axis. We often refer to the polar axis as the direct axis and the interpolar as the quadrature axis. This effect can be taken into account by resolving the armature current \( I_a \) into two components, one in time phase and the other in time quadrature with the excitation voltage as shown in Figure 3.16. The component \( I_d \) of the armature current is along the direct axis (the axis of the field poles), and the component \( I_q \) is along the quadrature axis.

Let us consider the effect of the direct-axis component alone. With \( I_d \) lagging the excitation EMF \( E_f \) by 90°, the resulting armature-reaction flux \( \phi_{ad} \) is directly opposite the filed poles as shown in Figure 3.17. The effect of the quadrature-axis component is to produce an armature-reaction flux \( \phi_{aq} \) which is in the quadrature-axis direction as shown in Figure 3.17. The phasor diagram with both components present is shown in Figure 3.18.

![Figure 3.16 Resolution of Armature Current in Two Components.](image1)

![Figure 3.17 Direct-Axis and Quadrature-Axis Air-Gap Fluxes in a Salient-Pole Synchronous Machine.](image2)
In the cylindrical rotor machine, we employed the synchronous reactance $x_s$ to account for the armature-reaction EMF in an equivalent circuit. The same argument can be extended to the salient-pole case. With each of the components currents $I_d$ and $I_q$, we associated component synchronous-reactance voltage drops, $jI_d x_d$ and $jI_q x_q$ respectively. The direct-axis synchronous reactance $x_d$ and the quadrature-axis synchronous reactance $x_q$ are given by

$$x_d = x_l + x_{q0}$$
$$x_q = x_l + x_{q0}$$

where $x_l$ is the armature leakage reactance and is assumed to be the same for direct-axis and quadrature-axis currents. The direct-axis and quadrature-axis magnetizing reactances $x_{q0}$ and $x_{q0}$ account for the inductive effects of the respective armature-reaction flux. Figure 3.19 shows a phasor diagram implementing the result.

$$E_f = V_r + I_a r_a + jI_d x_d + jI_q x_q$$  \hspace{1cm} (3.21)$$

In many instances, the power factor angle $\Phi$ at the machine terminals is explicitly known rather than the internal power factor angle $(\phi + \delta)$, which is required for the resolution of $I_a$ into its direct-axis and quadrature-axis components. We can avoid this difficulty by recalling that in phasor notation,

$$I_a = I_q + I_d$$  \hspace{1cm} (3.22)$$

Substitution of Eq. (3.22) into Eq. (3.21) for $I_a$ and rearranging, we obtain

$$E_f = V_r + I_a (r_a + j x_q) + jI_d (x_d - x_q)$$  \hspace{1cm} (3.23)$$

Let us define

$$E'_f = V_r + I_a (r_a + j x_q)$$  \hspace{1cm} (3.24)$$

Figure 3.18 Phasor Diagram for a Salient-Pole Synchronous Machine.
$E'_f$ as defined is in the same direction as $E_f$ since $jI_d$ is also along the same direction. Our procedure then is to obtain $E'_f$ as given by Eq. (3.24) and then obtain the component $I_d$ based on the phase angle of $E'_f$. Finally, we find $E_f$ as a result of

$$E_f = E'_f + jI_d(x_d - x_q) \quad (3.25)$$

This is shown in Figure 3.20.

**Example 3.5**

A 5-kVA, 220-V, Y-connected, three-phase, salient-pole synchronous generator is used to supply power to a unity PF load. The direct-axis synchronous reactance is 12 ohms and the quadrature-axis synchronous reactance is 7 ohms. Assume that rated current is delivered to the load at rated voltage and that armature resistance is negligible. Compute the excitation voltage and power angle.
Solution

\[ V_t = 127.02 \text{ V} \]
\[ I_a = \frac{5 \times 10^3}{220\sqrt{3}} = 13.12 \text{ A} \]

We calculate

\[ E'_f = V_t + jI_a x_q \]
\[ = 127.02 + j(13.12)(7) = 156.75 \angle 35.87^\circ \]

Moreover,

\[ I_d = I_a \sin 35.87 = 7.69 \text{ A} \]
\[ |E_f| = |E'_f| + |I_d(x_d - x_q)| \]
\[ = 156.75 + 7.69(12-7) = 195.20 \text{ V} \]
\[ \delta = 35.87^\circ \]

The following script uses MATLAB\textsuperscript{TM} to solve Example 3.5.

```matlab
% Example 3.5
% A 5 kVA, 220 Volts, Y connected, 3 phase, salient pole synchronous generator
PF=1;
VL=220;  % Volts
xd=12;
xq=7;
P=5*10^3;  % VA
Vt=VL/3^.5;
Ia=P/(VL*3^.5)
% We calculate
Ef_prime=Vt+i*Ia*xq;
abs(Ef_prime)
angle(Ef_prime)*180/pi
Id=Ia*sin(angle(Ef_prime));
Ef=abs(Ef_prime)+abs(Id*(xd-xq))
delta=angle(Ef_prime)*180/pi
```
The solution is

\[
\begin{align*}
\text{EDU»} \\
I_a &= 13.1216 \\
\text{ans} &= 156.7481 \\
\text{ans} &= 35.8722 \\
Ef &= 195.1931 \\
delta &= 35.8722
\end{align*}
\]

3.8 **SALIENT-POLE MACHINE POWER ANGLE CHARACTERISTICS**

The power angle characteristics for a salient-pole machine connected to an infinite bus of voltage \(V\) through a series reactance of \(x_e\) can be arrived at by considering the phasor diagram shown in Figure 3.21. The active power delivered to the bus is

\[
P = (I_d \sin \delta + I_q \cos \delta)V
\]

(3.26)

Similarly, the delivered reactive power \(Q\) is

\[
Q = (I_d \cos \delta - I_q \sin \delta)V
\]

(3.27)

To eliminate \(I_d\) and \(I_q\), we need the following identities obtained from inspection of the phasor diagram:

\[
I_d = \frac{E_f - V \cos \delta}{X_d}
\]

(3.28)

\[
I_q = \frac{V \sin \delta}{X_q}
\]

(3.29)

where

\[
X_d = x_d + x_e
\]

(3.30)

\[
X_q = x_q + x_e
\]

(3.31)

Substitution of Eqs. (3.28) and (3.29) into Eqs. (3.26) and (3.27) yields equations that contain six quantities – the two variables \(P\) and \(\delta\) and the four parameters \(E_f, V, X_d,\) and \(X_q\) – and can be written in many different ways. The following form illustrates the effect of saliency. Define \(P_d\) and \(Q_d\) as

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\[ P_d = \frac{V E_f}{X_d} \sin \delta \] (3.32)

and

\[ Q_d = \frac{V E_f}{X_d} \cos \delta - \frac{V^2}{X_d} \] (3.33)

The above equations give the active and reactive power generated by a round rotor machine with synchronous reactance \( X_d \). We thus have

\[ P = P_d + \frac{V^2}{2} \left( \frac{1}{X_q} - \frac{1}{X_d} \right) \sin 2\delta \] (3.34)

\[ Q = Q_d - V^2 \left( \frac{1}{X_q} - \frac{1}{X_d} \right) \sin^2 \delta \] (3.35)

The second term in the above two equations introduces the effect of salient poles, and in the power equation the term corresponds to reluctance.

Figure 3.21 A Salient-Pole Machine Connected to an Infinite Bus through an External Impedance.

Figure 3.22 Power Angle Characteristics of a Salient-Pole Synchronous Machine.
torque. Note that if $X_d = X_q$, as in a uniform air-gap machine, the second terms in both equations are zero. Figure 3.22 shows the power angle characteristics of a typical salient-pole machine.

The pull-out power and power angle $\delta$ for the salient-pole machine can be obtained by solving equation (3.36) requiring the partial derivative of $P$ with respect to $\delta$ to be equal to zero.

$$\frac{\partial P}{\partial \delta} = 0 \tag{3.36}$$

The actual value of pull-out power can be shown to be higher than that obtained assuming nonsaliency.

**Example 3.6**

A salient-pole synchronous machine is connected to an infinite bus through a link with reactance of 0.2 p.u. The direct-axis and quadrature-axis reactances of the machine are 0.9 and 0.65 p.u., respectively. The excitation voltage is 1.3 p.u., and the voltage of the infinite bus is maintained at 1 p.u. For a power angle of $30^\circ$, compute the active and reactive power supplied to the bus.

**Solution**

We calculate $X_d$ and $X_q$ as

$$X_d = x_d + x_e = 0.9 + 0.2 = 1.1$$

$$X_q = x_q + x_e = 0.65 + 0.2 = 0.85$$

Therefore,

$$P = \frac{(1.3)(1)}{1.1} \sin 30^\circ + \frac{1}{2} \left( \frac{1}{0.85} - \frac{1}{1.1} \right) \sin 60^\circ$$

$$= 0.7067 \text{ p.u.}$$

Similarly, the reactive power is obtained using Eq. (3.32) as:

$$Q = \frac{(1.3)(1)}{1.1} \cos 30^\circ - \left( \frac{\cos^2 30^\circ}{1.1} + \frac{\sin^2 30^\circ}{0.85} \right)$$

$$= 0.0475 \text{ p.u.}$$

**PROBLEMS**

**Problem 3.1**

A 5-k VA, 220-V, 60-Hz, six-pole, Y-connected synchronous generator has a leakage reactance per phase of 0.78 ohms and negligible armature resistance. The armature-reaction EMF for this machine is related to the armature current
by

\[ E_{ar} = -j16.88(I_a) \]

Assume that the generated EMF is related to field current by

\[ E_f = 25I_f \]

A. Compute the field current required to establish rated voltage across the terminals of a unity power factor load that draws rated generator armature current.

B. Determine the field current needed to provide rated terminal voltage to a load that draws 125 percent of rated current at 0.8 PF lagging.

**Problem 3.2**
A 9375 kVA, 13,800 kV, 60 Hz, two pole, Y-connected synchronous generator is delivering rated current at rated voltage and unity PF. Find the armature resistance and synchronous reactance given that the filed excitation voltage is 11935.44 V and leads the terminal voltage by an angle 47.96°.

**Problem 3.3**
The magnitude of the field excitation voltage for the generator of Problem (3.2) is maintained constant at the value specified above. Find the terminal voltage when the generator is delivering rated current at 0.8 PF lagging.

**Problem 3.4**
A 180 kVA, three-phase, Y-connected, 440 V, 60 Hz synchronous generator has a synchronous reactance of 1.6 ohms and a negligible armature resistance. Find the full load generated voltage per phase at 0.8 PF lagging.

**Problem 3.5**
The synchronous reactance of a cylindrical rotor synchronous generator is 0.9 p.u. If the machine is delivering active power of 1 p.u. to an infinite bus whose voltage is 1 p.u. at unity PF, calculate the excitation voltage and the power angle.

**Problem 3.6**
The synchronous reactance of a cylindrical rotor machine is 1.2 p.u. The machine is connected to an infinite bus whose voltage is 1 p.u. through an equivalent reactance of 0.3 p.u. For a power output of 0.7 p.u., the power angle is found to be 30°.

A. Find the excitation voltage \( E_f \) and the pull-out power.

B. For the same power output the power angle is to be reduced to 25°. Find the value of the reduced equivalent reactance connecting the machine to the bus to achieve this. What would be the new pull-out power?
Problem 3.7
Solve Problem 3.5 using MATLAB™.

Problem 3.8
A cylindrical rotor machine is delivering active power of 0.8 p.u. and reactive power of 0.6 p.u. at a terminal voltage of 1 p.u. If the power angle is 20°, compute the excitation voltage and the machine’s synchronous reactance.

Problem 3.9
A cylindrical rotor machine is delivering active power of 0.8 p.u. and reactive power of 0.6 p.u. when the excitation voltage is 1.2 p.u. and the power angle is 25°. Find the terminal voltage and synchronous reactance of the machine.

Problem 3.10
A cylindrical rotor machine is supplying a load of 0.8 PF lagging at an infinite bus. The ratio of the excitation voltage to the infinite bus voltage is found to be 1.25. Compute the power angle $\delta$.

Problem 3.11
The synchronous reactance of a cylindrical rotor machine is 0.8 p.u. The machine is connected to an infinite bus through two parallel identical transmission links with reactance of 0.4 p.u. each. The excitation voltage is 1.4 p.u. and the machine is supplying a load of 0.8 p.u.

A. Compute the power angle $\delta$ for the outlined conditions.
B. If one link is opened with the excitation voltage maintained at 1.4 p.u. Find the new power angle to supply the same load as in (a).

Problem 3.12
The synchronous reactance of a cylindrical rotor generator is 1 p.u. and its terminal voltage is 1 p.u. when connected to an infinite bus through a reactance 0.4 p.u. Find the minimum permissible output vars for zero output active power and unity output active power.

Problem 3.13
The apparent power delivered by a cylindrical rotor synchronous machine to an infinite bus is 1.2 p.u. The excitation voltage is 1.3 p.u. and the power angle is 20°. Compute the synchronous reactance of the machine, given that the infinite bus voltage is 1 p.u.

Problem 3.14
The synchronous reactance of a cylindrical rotor machine is 0.9 p.u. The machine is connected to an infinite bus through two parallel identical transmission links with reactance of 0.6 p.u. each. The excitation voltage is 1.5 p.u., and the machine is supplying a load of 0.8 p.u.

A. Compute the power angle $\delta$ for the given conditions.
B. If one link is opened with the excitation voltage maintained at 1.5
p.u., find the new power angle to supply the same load as in part (a).

Problem 3.15
The reactances $x_d$ and $x_q$ of a salient-pole synchronous generator are 0.95 and 0.7 per unit, respectively. The armature resistance is negligible. The generator delivers rated kVA at unity PF and rated terminal voltage. Calculate the excitation voltage.

Problem 3.16
The machine of Problem 3.15 is connected to an infinite bus through a link with reactance of 0.2 p.u. The excitation voltage is 1.3 p.u. and the infinite bus voltage is maintained at 1 p.u. For a power angle of 25°, compute the active and reactive power supplied to the bus.

Problem 3.17
A salient pole machine supplies a load of 1.2 p.u. at unity power factor to an infinite bus whose voltage is maintained at 1.05 p.u. The machine excitation voltage is computed to be 1.4 p.u. when the power angle is 25°. Evaluate the direct-axis and quadrature-axis synchronous reactances.

Problem 3.18
Solve Problem 3.17 using MATLAB™.

Problem 3.19
The reactances $x_d$ and $x_q$ of a salient-pole synchronous generator are 1.00 and 0.6 per unit respectively. The excitation voltage is 1.77 p.u. and the infinite bus voltage is maintained at 1 p.u. For a power angle of 19.4°, compute the active and reactive power supplied to the bus.

Problem 3.20
For the machine of Problem 3.17, assume that the active power supplied to the bus is 0.8 p.u. compute the power angle and the reactive power supplied to the bus. (Hint: assume $\cos \delta \equiv 1$ for an approximation).